Stability of Shallow Tunnels in Soils Using Analytical and Numerical Methods

by

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Stability of shallow circular tunnels in soils using analytical and numerical models

C. Carranza-Torres, T. Reich & D. Saftner
Department of Civil Engineering, University of Minnesota, Duluth Campus, Minnesota, USA

ABSTRACT: This paper revisits a classical problem of geotechnical engineering involving the stability of shallow circular tunnels excavated in frictional cohesive materials. The problem is of practical interest since, among others, it allows establishing conditions of stability for the front of tunnels in soils excavated manually or using mechanized methods. A historical background of computational methods developed to establish the stability conditions of shallow cavities in soils is presented first. In particular, analytical models based on lower and upper bound theories of plasticity are discussed. Thereafter a classical lower bound model due to Caquot is analyzed and extended to account for the presence of a surface surcharge and water in the soil being excavated. This model is proposed as a means of getting a first estimate of the stability conditions of shallow tunnels under various hydraulic conditions, using a closed-form solution. The concept of factor of safety, traditionally used in the assessment of stability of slopes in frictional cohesive materials, is also included in the model. Results obtained with the extended Caquot's model are shown to be in accordance with those obtained with more sophisticated finite element and finite difference methods. A computer spreadsheet including the implementation of Caquot's extended solution is also provided in the paper.

1 INTRODUCTION

The stability of shallow circular cavities excavated in soils is an i
nificance in geotechnical

Stability of shallow circular tunnels in soils using analytical and numerical models

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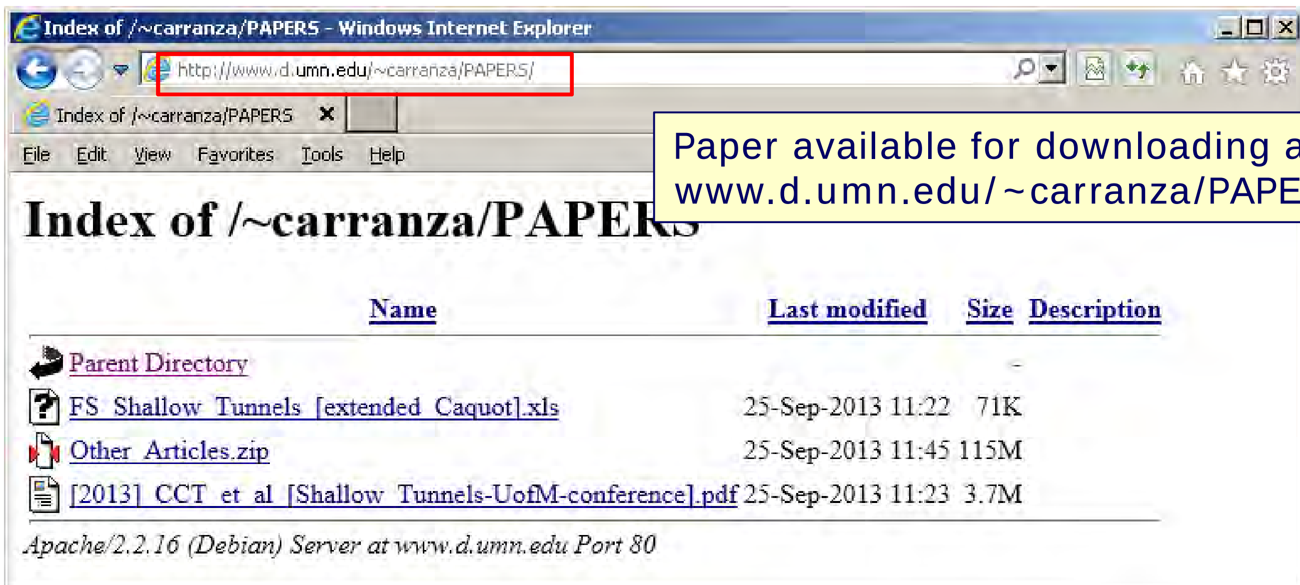

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1 INTRODUCTION

The stability of shallow circular cavities excavated in soils is an i...
nificance in geotech...


UNIVERSITY OF MINNESOTA
60th Annual Geotechnical Engineering Conference
February 24, 2012
Tyler Reich
(MSc student)
David Saftner
(faculty)

for the design.

Finally, the constitutive model for the soil considered in this paper for the tunnel problems was the simple Mohr-Coulomb failure criterion. Most commercial software implementing the strength reduction technique allows application of other constitutive models. A popular one is the Hoek-Brown failure criterion (Hoek & Brown 1980; Hoek et al. 2002), which is widely used nowadays in design of excavations in rock masses. The analysis presented in this paper (with the additional improvements discussed above) could be extended for the case of weak rocks that satisfy the Hoek-Brown failure criterion.

ACKNOWLEDGEMENTS

Many of the developments presented in this paper have been carried out by the first author while working with Geodata S.p.A. in Turin, Italy, during a two-month working visit in 2004 (see Carranza-Torres 2004). These initial developments are serving as a basis for research work conducted by the second author of this paper for his MSc graduate work, which is being supervised by the first and third authors. The first author would like to thank Geodata's president, Dr. Piergiorgio Grasso, and his associates, Dr. Giordano Russo and Dr. Shulin Xu, for the support and hospitality provided while working with their group in 2004.

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Atkinson, J. H. & Potts, D. M. 1977. Stability of a shallow circular tunnel in cohesionless soil. *Geotechnique 27*(2), 203–215.

Broms, B. B. & Bennemark, H. 1967. ...

Structure of this presentation

- Shallow tunnel collapses.
- Analytical and numerical models for the analysis of stability of shallow tunnels.
- A proposed analytical model for analyzing stability of shallow tunnels.
- Comparison of results with proposed analytical and numerical models.
- Scaling of factor of safety results.
- Final comments.

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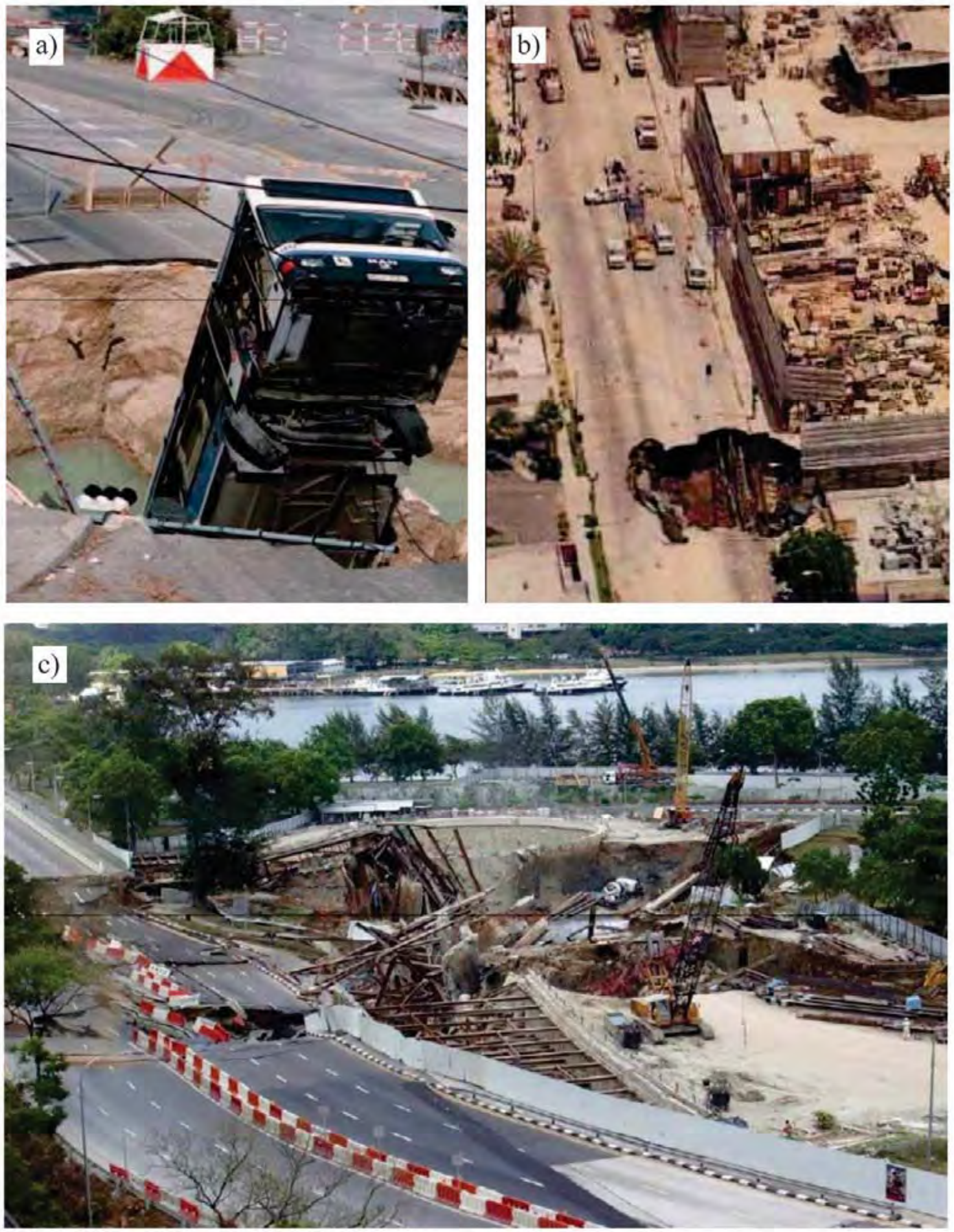


a) Collapse during construction of the Munich Metro (after Construction Today, 1994a).

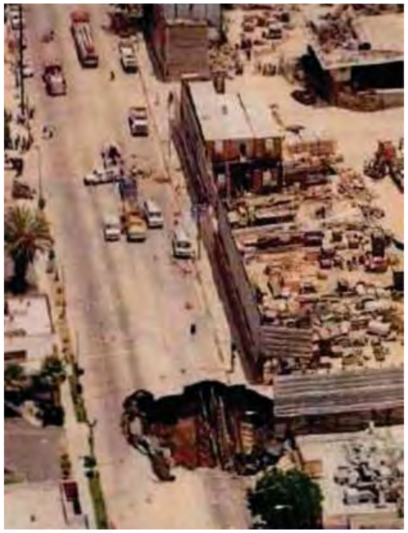
b) Collapse during construction of the LA Metro (after Civil Engineer International, 1995).

c) Collapse during construction of the Singapore underground Mass Rapid Transit (MRT) system (after Government of Singapore, 2005).

The metro tunnel collapse at Hollywood Boulevard, Los Angeles

- It occurred on June 22, 1995, while re-mining an existing tunnel of 6.7 m diameter, excavated with TBM, at 25 m below the ground surface (the re-mining work was intended to correct a tunnel alignment problem).

- Ground conditions included hard siltstone overlain by alluvium with groundwater level located 11 meters below the ground surface.

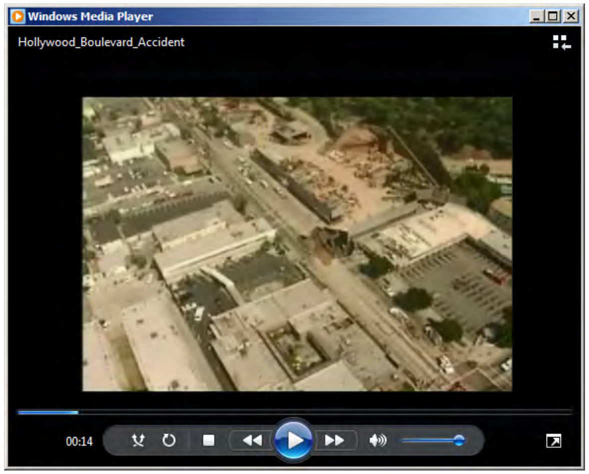


Sources: Civil Engineering International, 1995. History Channel Series “The Best of Modern Marvels”, volume 3, “Engineering Disasters”, 14, “Hollywood Boulevard”.

The metro tunnel collapse at Hollywood Boulevard, Los Angeles

- The collapse produced a 25 m deep sinkhole that started to fill rapidly with fluid from broken water and sewage lines, eventually breaking through the tunnel (flooding the tunnel system).

- The cause of the collapse was reduction of tunnel support pressure by removal of temporary steel sections installed during re-mining.



Sources: Civil Engineering International, 1995. History Channel Series “The Best of Modern Marvels”, volume 3, “Engineering Disasters”, 14, “Hollywood Boulevard”.

Collapses at the tunnel front. The Munich Metro Collapse.

- It occurred on June 22, 1995, while excavating a 7 m diameter tunnel, at a depth of approximately 18 m, following NATM (New Austrial Tunnelling Method) technique, using road-header and sprayed concrete lining as support.

- Ground conditions included stratum of gravel (with a phreatic surface), overlying a relatively impermeable marl stratum (where tunnel was excavated).

- The failure involved formation of a cave at the tunnel front.

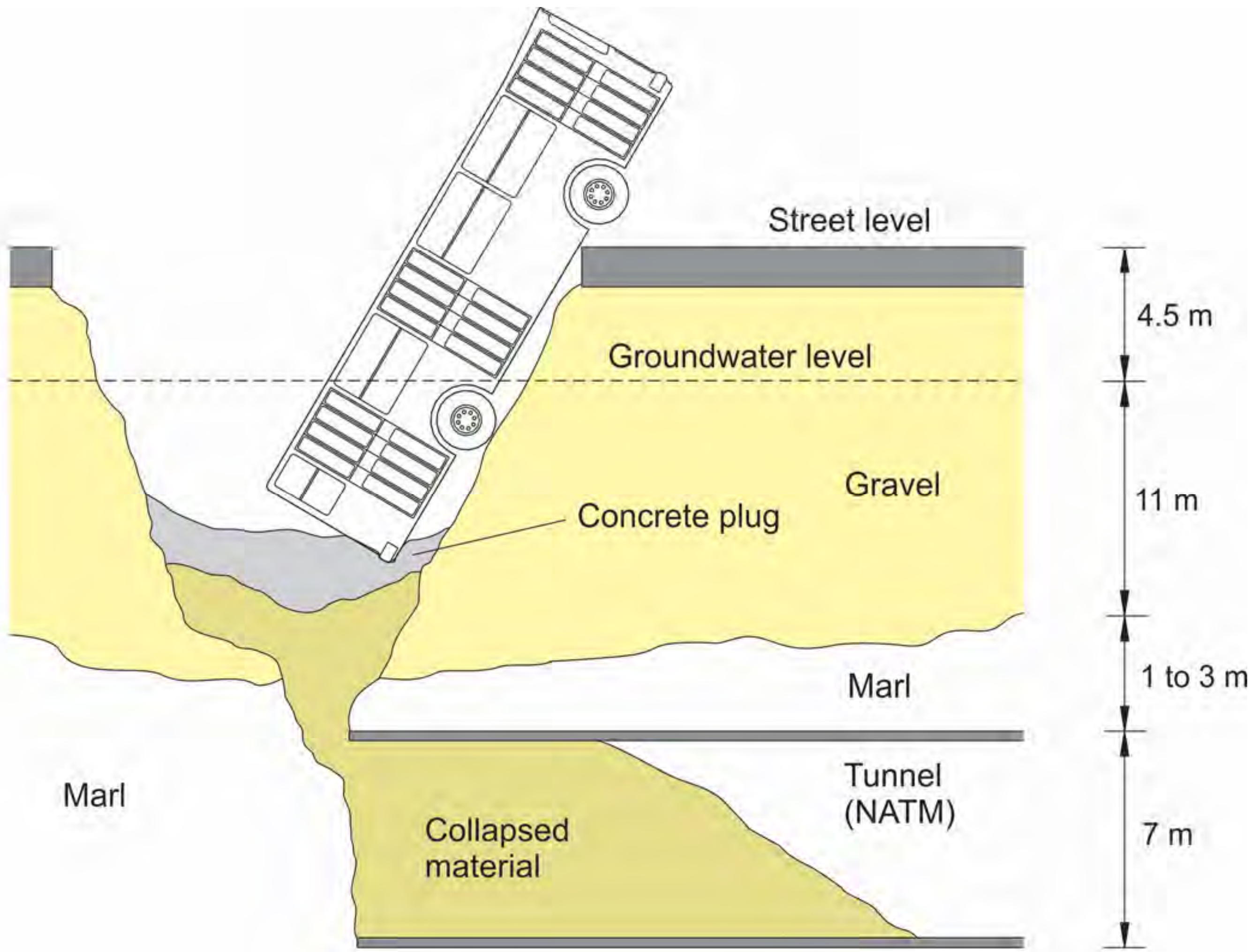
Sources: Construction Today (1994a, 1994b).

The metro tunnel collapse at Hollywood Boulevard, Los Angeles

- The forensic engineer in the video (Dr. Wolfgang Roth, from URS, LA), states "...*they took away the closed ring capacity of the liner*...".

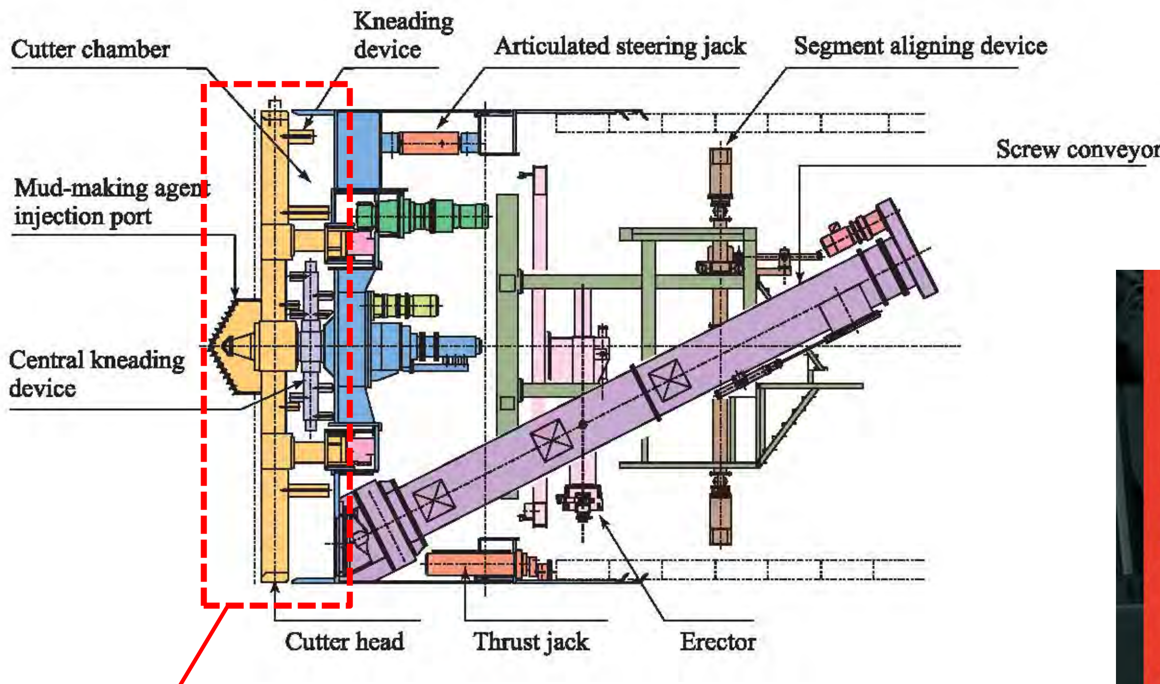
- The statement above can be reworded as "the collapse occurred for not *closing the circle* of support continuity".

Sources: Civil Engineering International, 1995. History Channel Series "The Best of Modern Marvels", volume 3, "Engineering Disasters", 14, "Hollywood Boulevard".



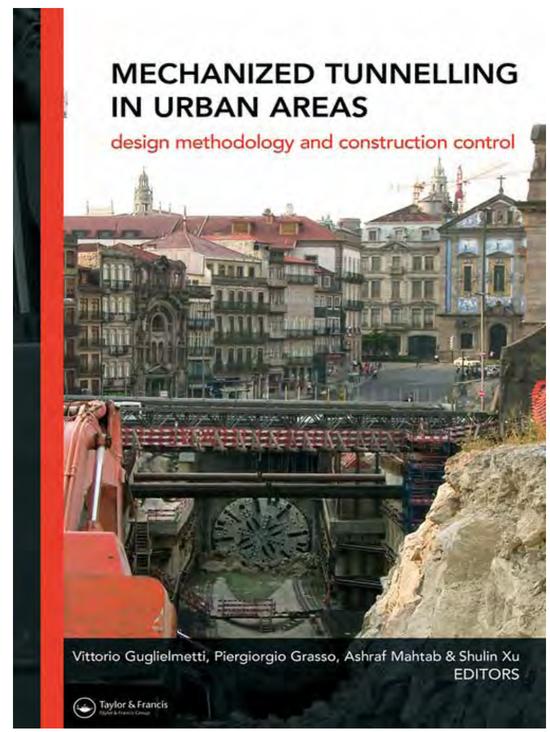
Sources: Construction Today (1994a, 1994b).

Required pressure at the front when using mechanized methods (e.g., with a Earth Pressure Balance Machine)



mud at pressure

Sources: Guglielmetti et al. (2008) ‘Mechanized Tunnelling in Urban Areas’ and ‘Hitachi Shield Machines’, Hitachi Construction Machinery Co., Ltd., www.hitachi-c-m.com



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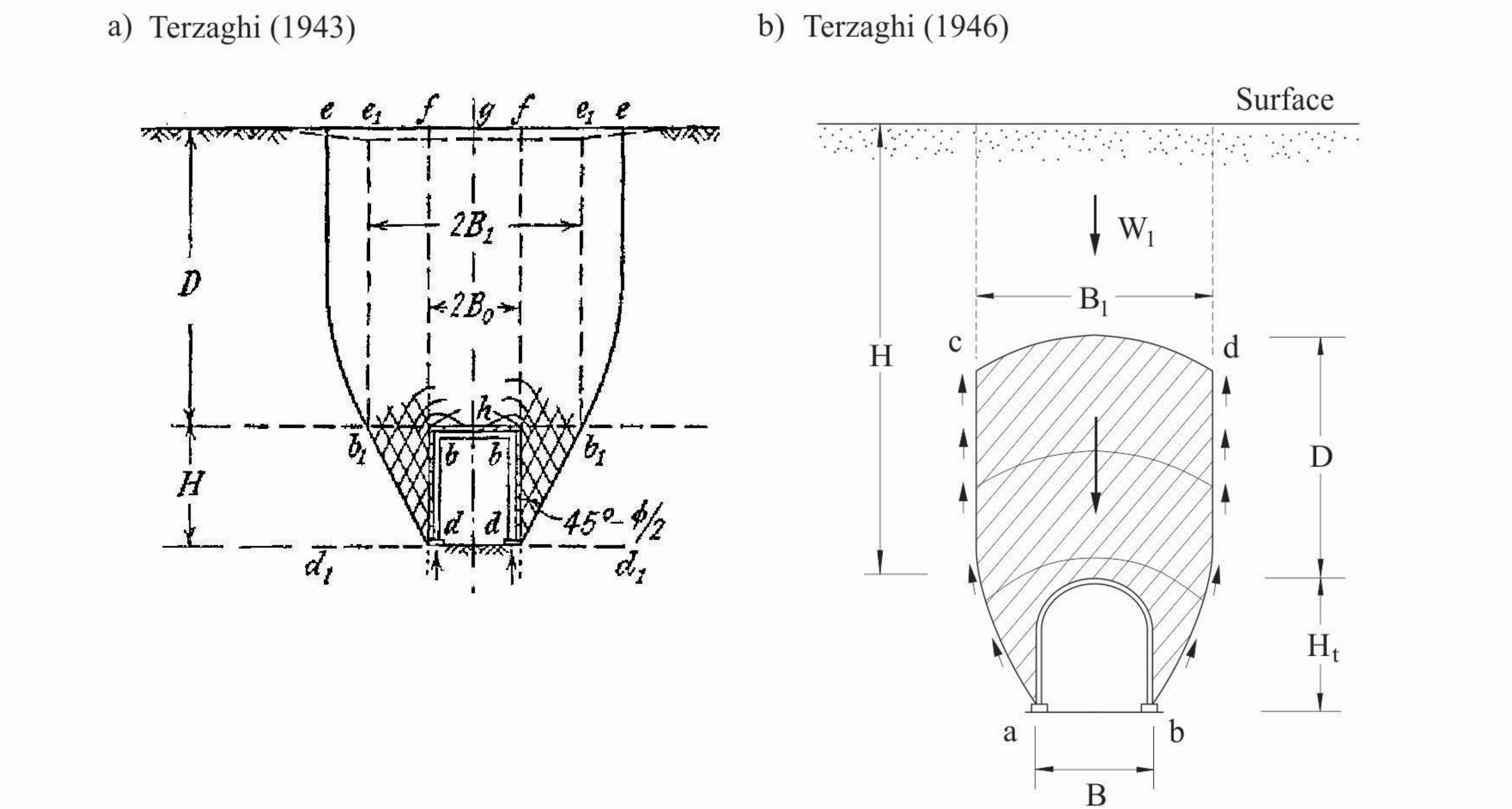
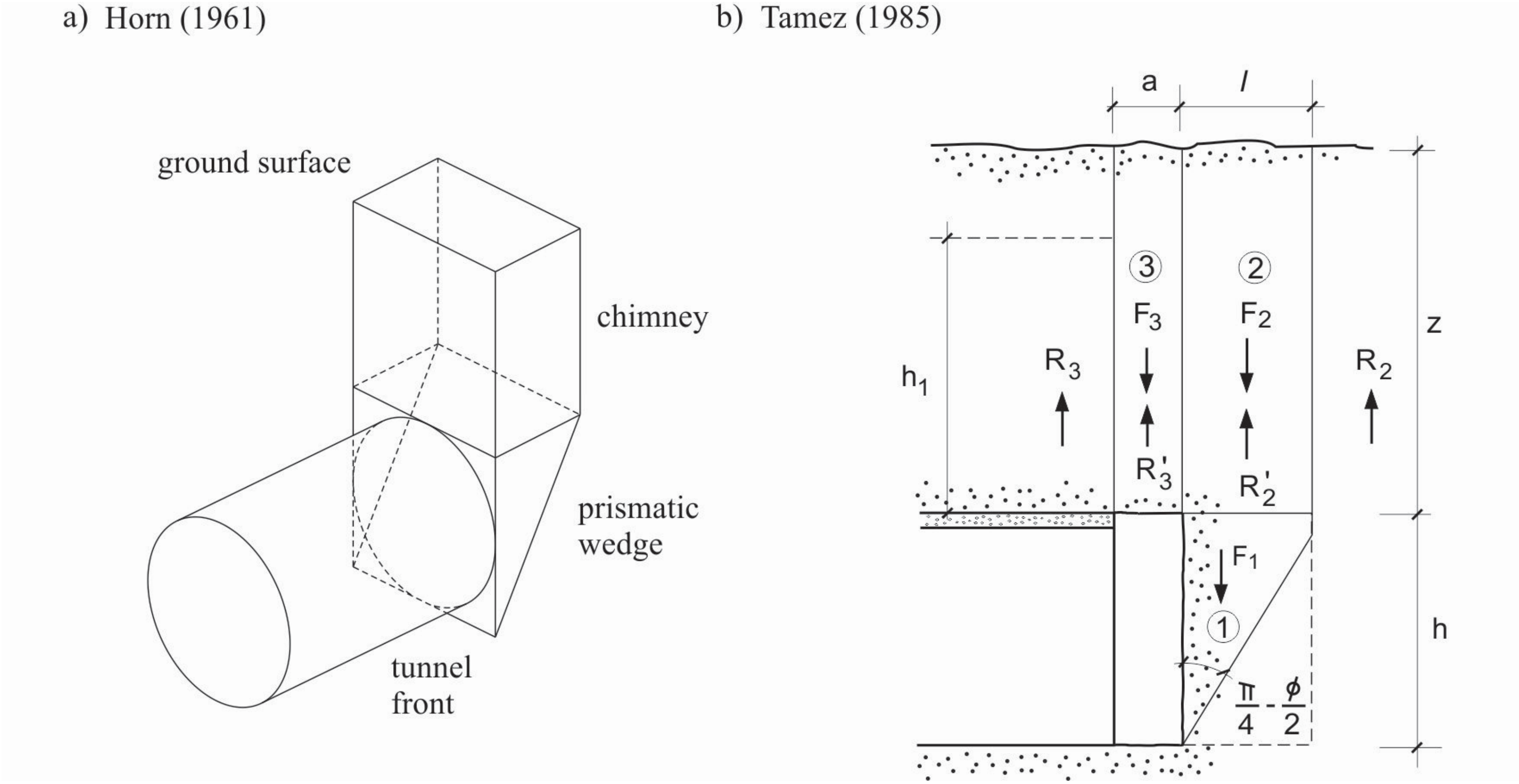
Analytical models for stability of shallow tunnels



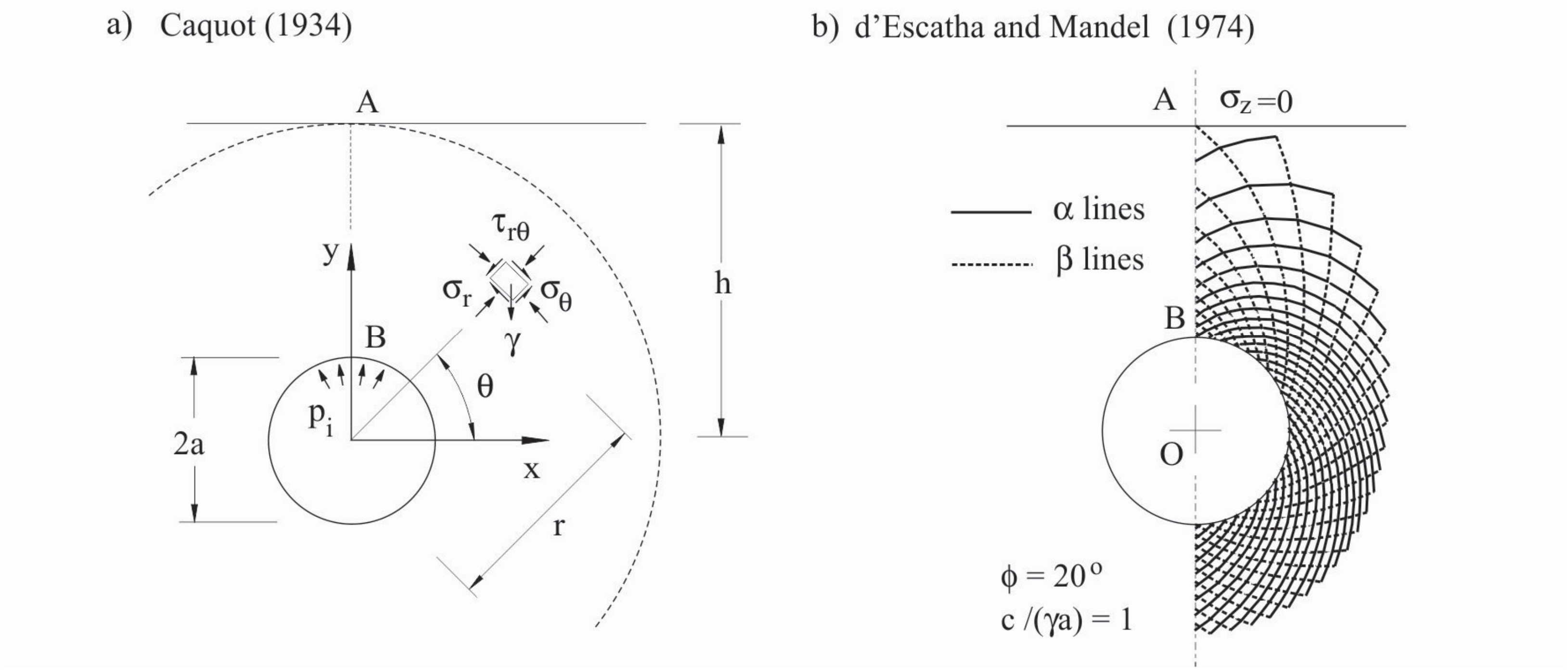
Sources: a) Terzaghi, K. 1943. Theoretical Soil Mechanics. New York: John Wiley & Sons.
b) Proctor, R. V. & White, T. L. 1946. Rock Tunnelling with Steel Supports. Commercial Shearing, Inc., Ohio.

Analytical models for stability of shallow tunnels



Sources: a) Horn, M. 1961. Horizontal earth pressure on vertical tunnel fronts. National Conference of Hungarian Civil Engineering Industry. Translation into German by STUVA, Düsseldorf.
b) Cornejo, L. 1989. Instability at the face: its repercussions for tunnelling technology. Tunnels and Tunnelling, 69–74.

Analytical models for stability of shallow tunnels



Sources: a) Caquot, A. 1934. Équilibre des massifs a frottement interne. Paris: Gauthier-Villars. b) d'Escatha,Y. & Mandel, J. 1974. Stabilité d'une galerie peu profonde en terrain meuble. *Industrie Minérale 6,* 1–9.

APPENDIX A. DEMONSTRATION OF CAQUOT'S STABILITY EQUATION

Caquot's solution (Caquot 1934; Caquot & Kerisel 1949) is a classical solution for determining the stress field around a circular tunnel located below a flat surface. This appendix presents a demonstration of equation (1) in the main text, which is one of the fundamental expressions conforming Caquot's solution. The analysis that follows refer to the same problem presented in Figure 9.

In reference to Figure 9, and to simplify the formulation, the radial distance, r, is first scaled with respect to the tunnel radius, a. This defines the dimensionless ratio, ρ, i.e.,

$$\rho = \frac{r}{a} \tag{A-1}$$

Note that according to equation (A-1), the position of the ground surface, $r = h$ in Figure 9, is determined by the variable ξ, i.e.,

$$\xi = \frac{h}{a} \tag{A-2}$$

The material surrounding the tunnel is assumed to obey the Mohr-Coulomb failure criterion, so that the relationship between major and minor principal stresses at failure, the quantities σ_1 and σ_3, respectively, is

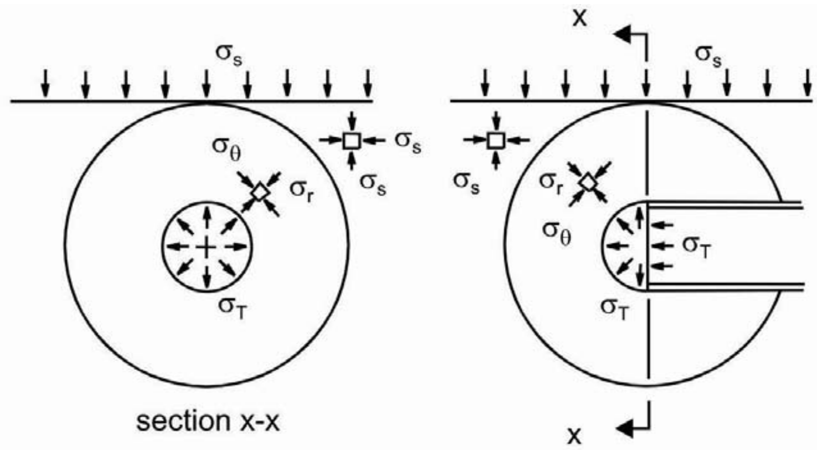
$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi} \tag{A-3}$$

In equation (A-3), c is the cohesion and N_ϕ is the passive reaction coefficient of the material (see, for example, Terzaghi et al. 1996), which is computed from the internal friction angle, ϕ, as follows

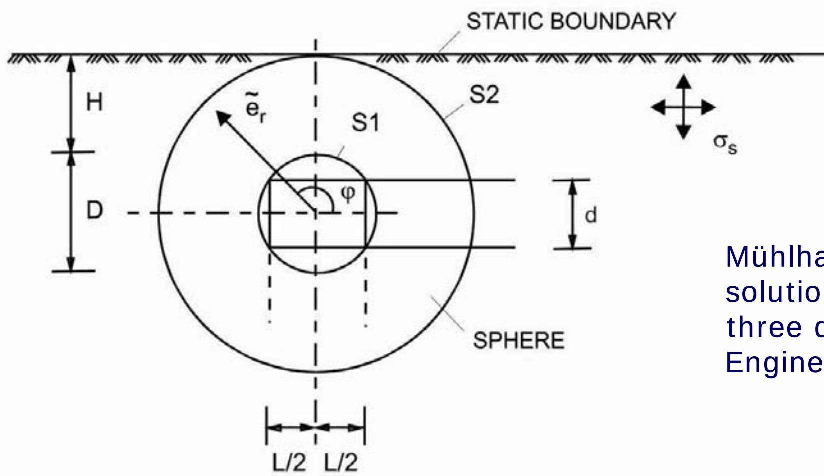
$$N_\phi = \frac{1+\sin\phi}{1-\sin\phi} = \tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right) \tag{A-4}$$

Also, to simplify the formulation, a particular form of scaling that applies to Mohr-Coulomb shear failure will be used. The scaling consists in adding the term $c\tan\phi$, or equivalently, the term $2c\sqrt{N_\phi}/(N_\phi - 1)$ to the normal stresses (see, for example, Anagnostou & Kovari 1993; Carranza-Torres 2003). Therefore, when scaled according to the rule mentioned above, the radial and hoop stresses, σ_r and σ_θ, respectively (see Figure 9), become

Analytical models for stability of shallow tunnels



Davis, E. H., Gunn, M. J., Mair, R. J. & Seneviratne, H. N. 1980. The stability of shallow tunnels and underground openings in cohesive material. Geotechnique 30(4), 397–416.



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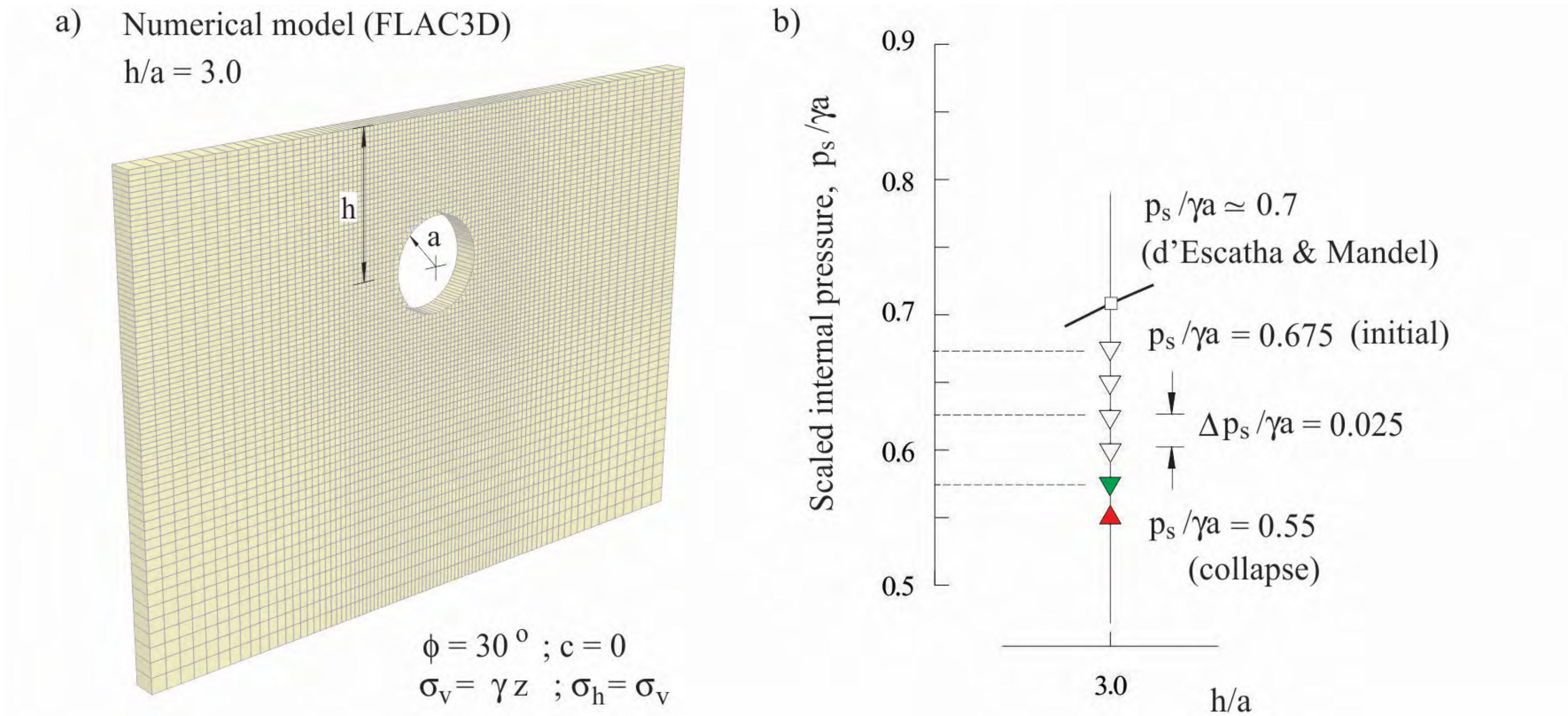
Numerical models for stability of shallow tunnels



Source: Fairhurst, C. & Carranza-Torres, C. 2002. *"Closing the Circle"*. In J. Labuz & J. Bentler (Eds.), Proceedings of the 50th Annual Geotechnical Engineering Conference. February 22, 2002. University of Minnesota. (Available for downloading at 'Fairhurst Files', www.itascacg.com).

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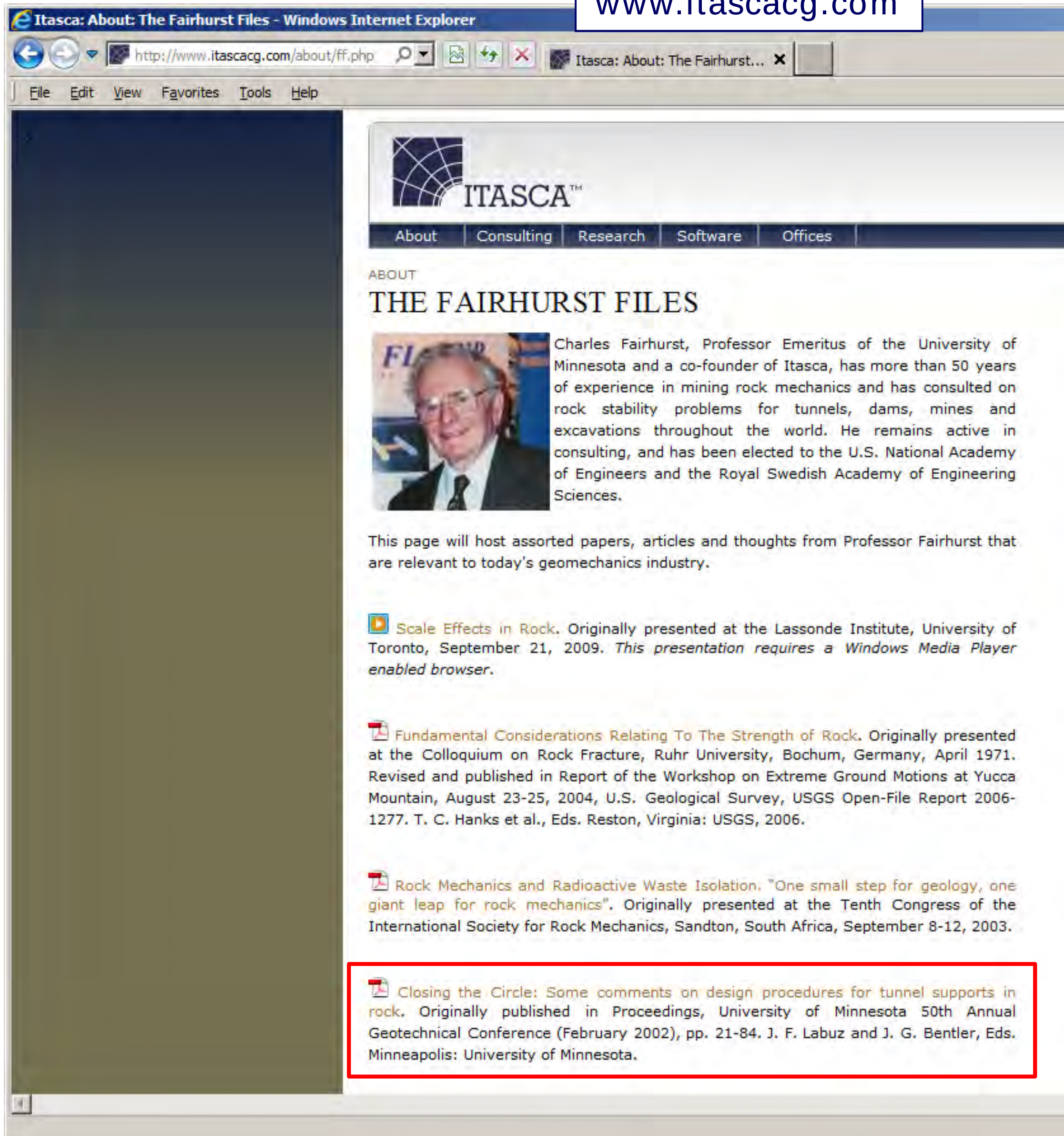

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ABOUT

THE FAIRHURST FILES

Charles Fairhurst, Professor Emeritus of the University of Minnesota and a co-founder of Itasca, has more than 50 years of experience in mining rock mechanics and has consulted on rock stability problems for tunnels, dams, mines and excavations throughout the world. He remains active in consulting, and has been elected to the U.S. National Academy of Engineers and the Royal Swedish Academy of Engineering Sciences.

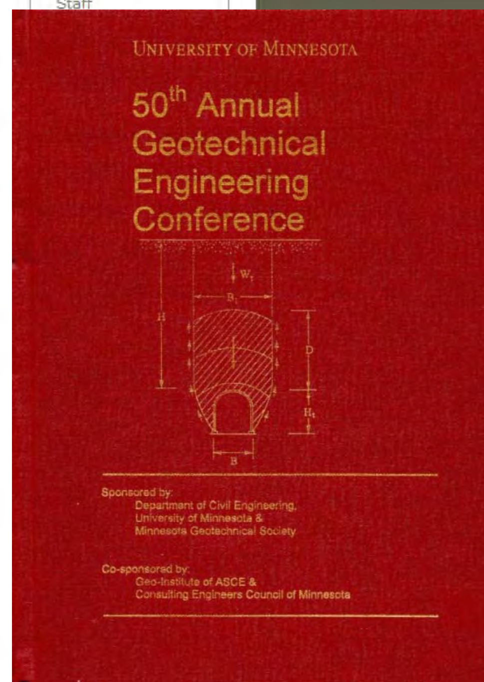
This page will host assorted papers, articles and thoughts from Professor Fairhurst that are relevant to today's geomechanics industry.

Scale Effects in Rock. Originally presented at the Lassonde Institute, University of Toronto, September 21, 2009. *This presentation requires a Windows Media Player enabled browser.*

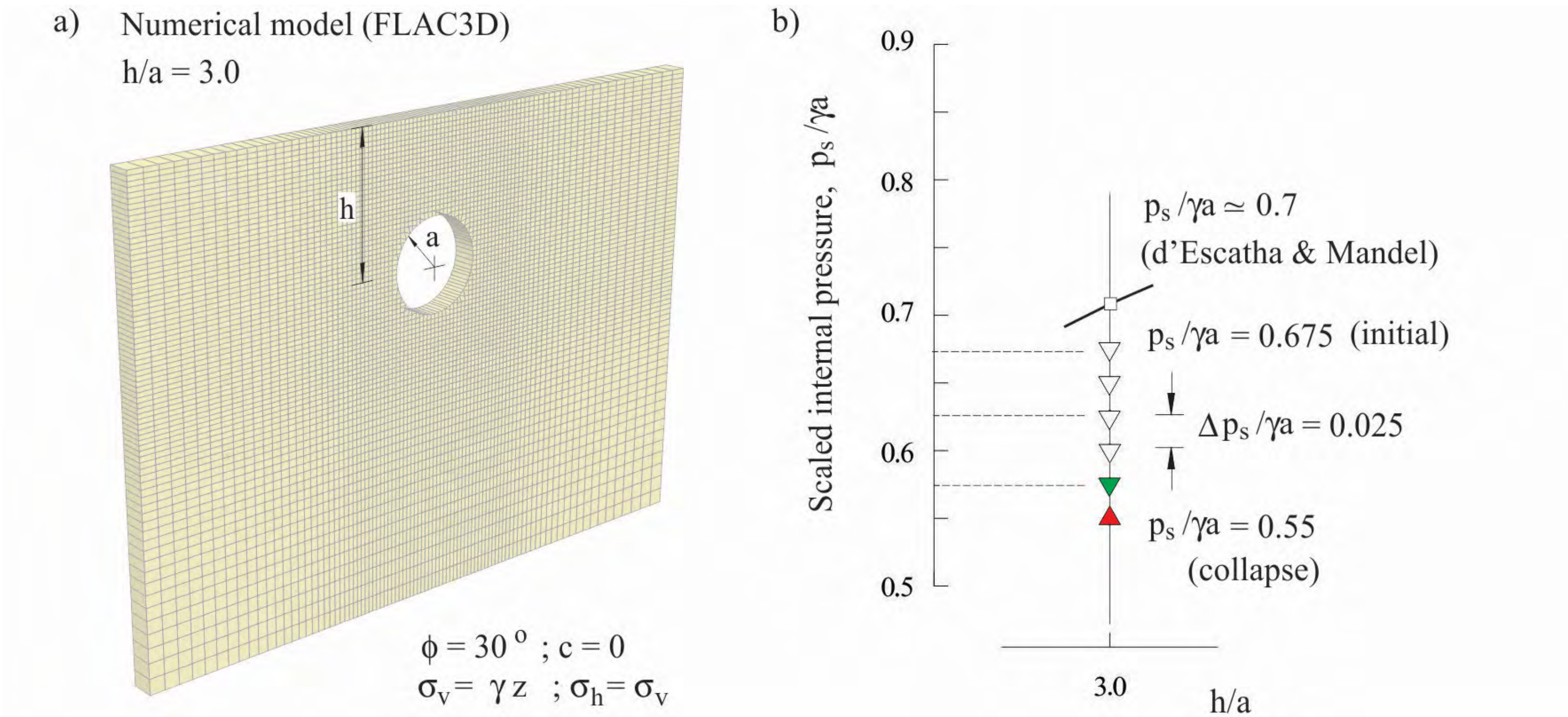
Fundamental Considerations Relating To The Strength of Rock. Originally presented at the Colloquium on Rock Fracture, Ruhr University, Bochum, Germany, April 1971. Revised and published in Report of the Workshop on Extreme Ground Motions at Yucca Mountain, August 23-25, 2004, U.S. Geological Survey, USGS Open-File Report 2006-1277. T. C. Hanks et al., Eds. Reston, Virginia: USGS, 2006.

Rock Mechanics and Radioactive Waste Isolation. "One small step for geology, one giant leap for rock mechanics". Originally presented at the Tenth Congress of the International Society for Rock Mechanics, Sandton, South Africa, September 8-12, 2003.

Closing the Circle: Some comments on design procedures for tunnel supports in rock. Originally published in Proceedings, University of Minnesota 50th Annual Geotechnical Conference (February 2002), pp. 21-84. J. F. Labuz and J. G. Bentler, Eds. Minneapolis: University of Minnesota.

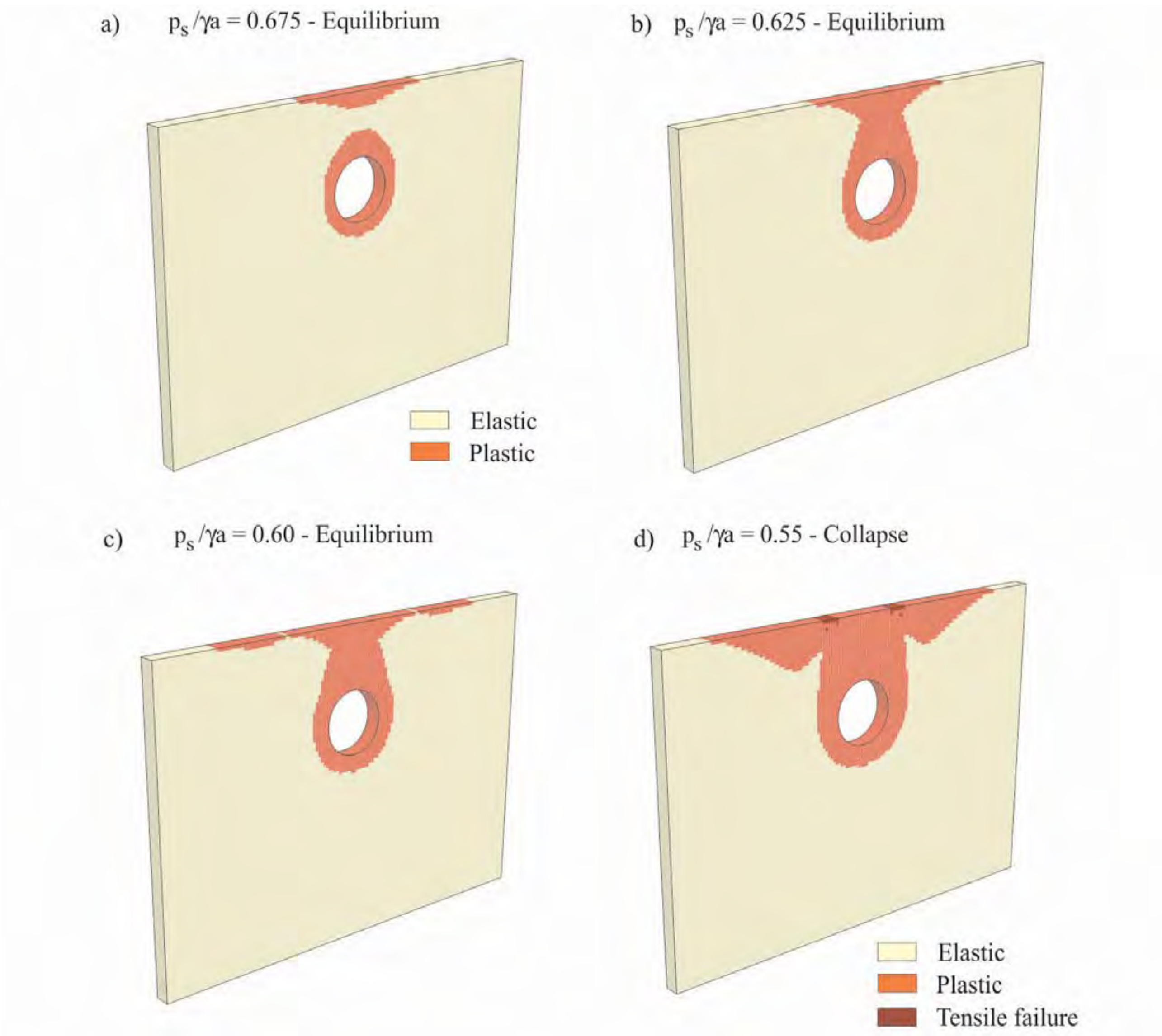


Numerical models for stability of shallow tunnels



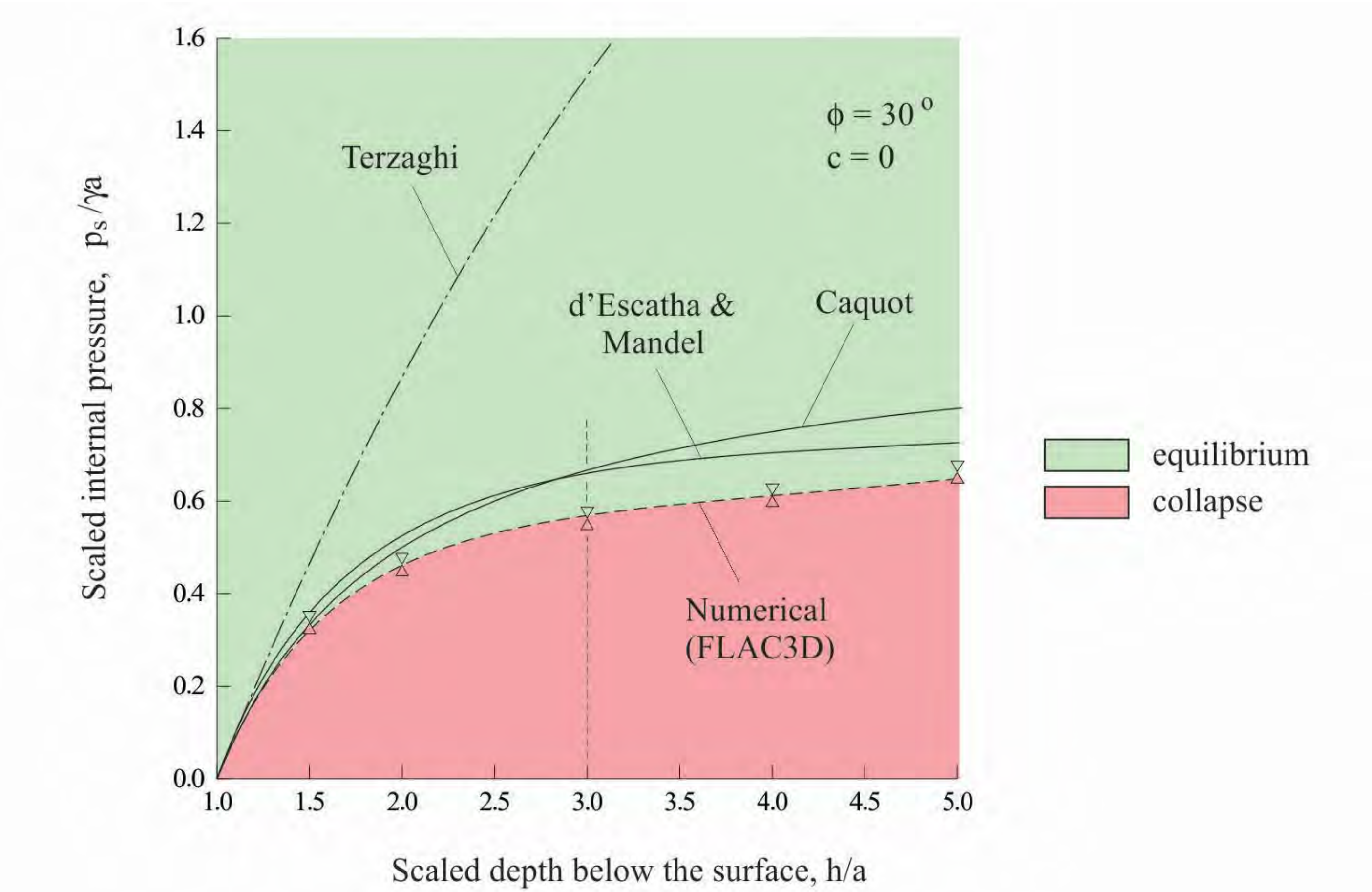
Source: Fairhurst, C. & Carranza-Torres, C. 2002. *"Closing the Circle"*. In J. Labuz & J. Bentler (Eds.), Proceedings of the 50th Annual Geotechnical Engineering Conference. February 22, 2002. University of Minnesota. (Available for downloading at 'Fairhurst Files', www.itascacg.com).

Numerical models for stability of shallow tunnels



Source: Fairhurst and Carranza-Torres (2002) *"Closing the Circle"*

Numerical models for stability of shallow tunnels



Source: Fairhurst and Carranza-Torres (2002) *"Closing the Circle"*

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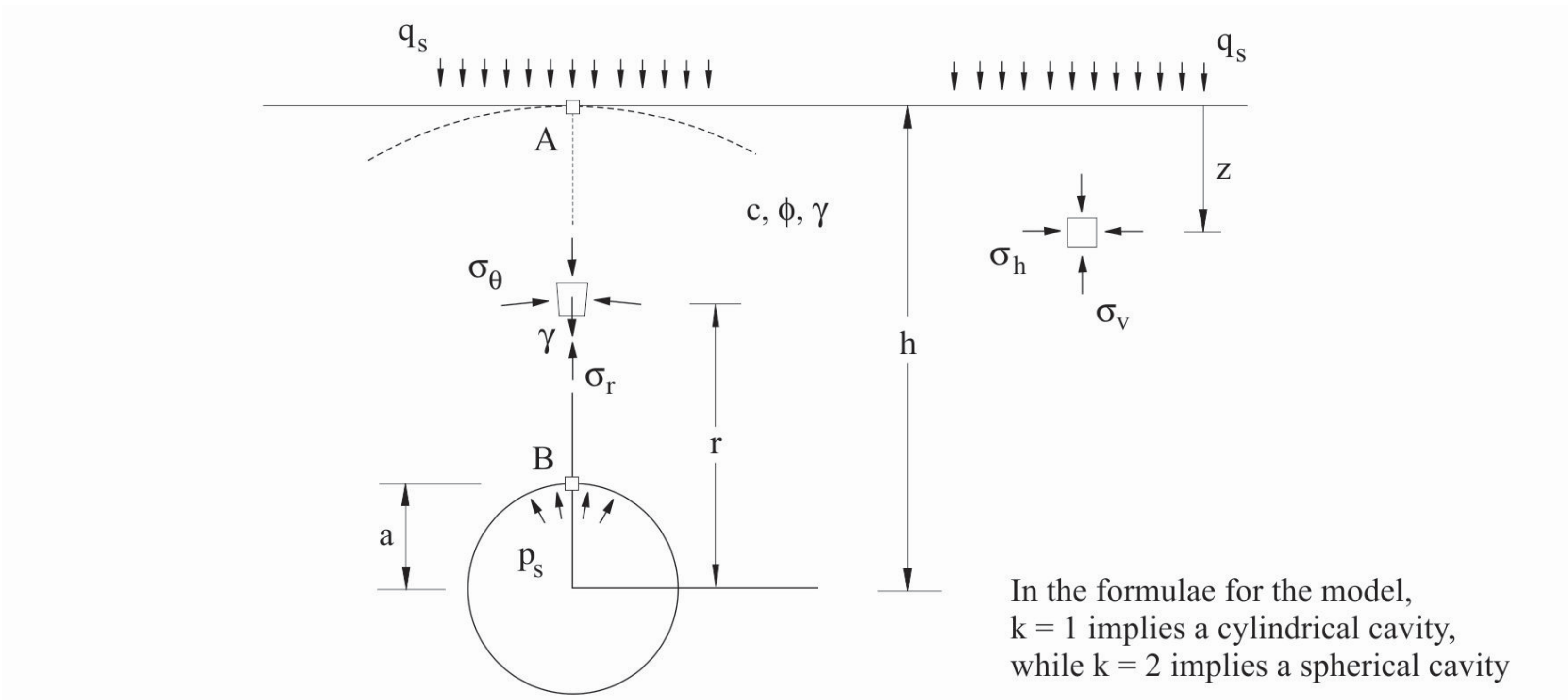
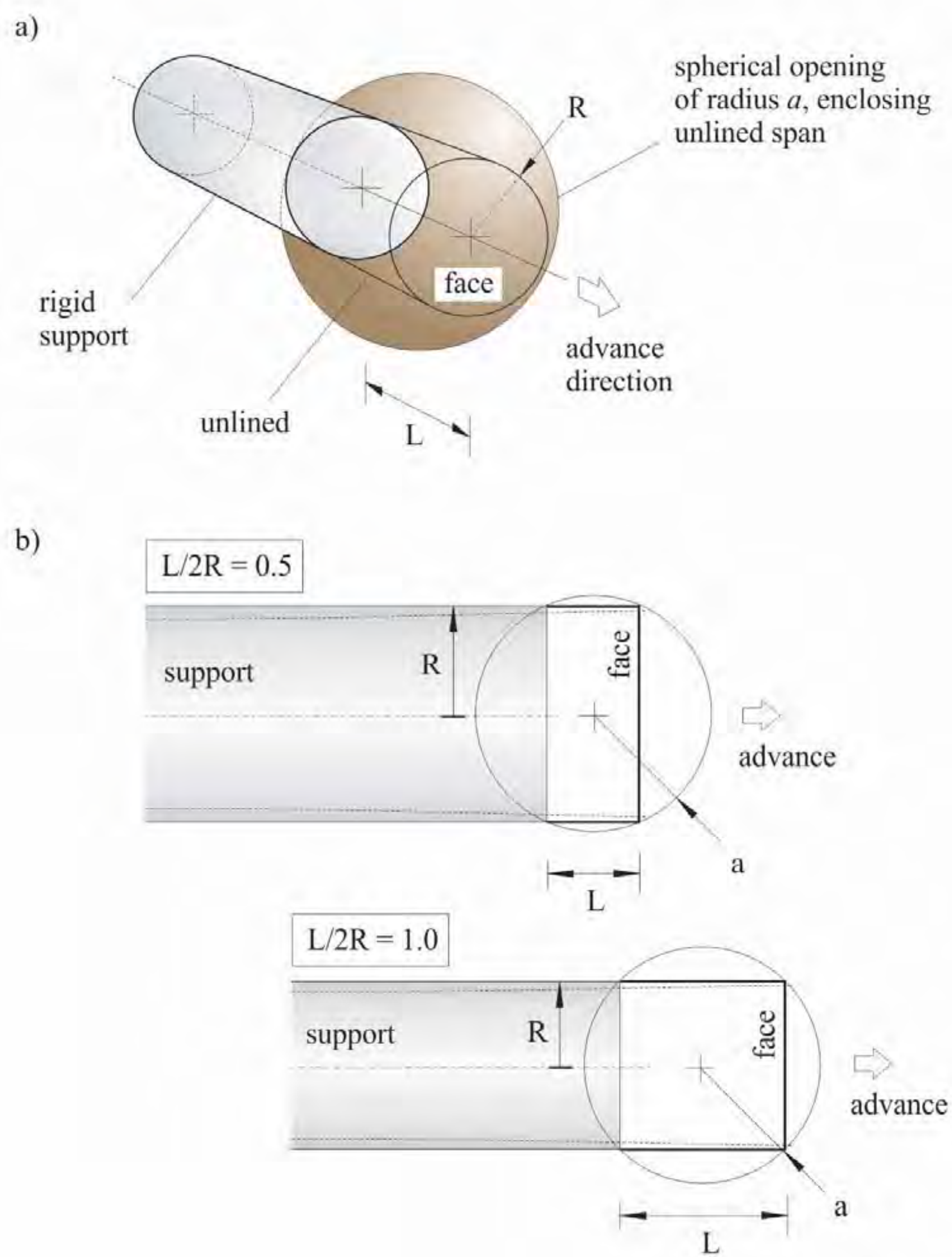
Basic characteristics of the proposed model for analysis of shallow tunnel stability (as described in Sections 4, 5 and the appendices in the paper)

- Use of a generalized form of Caquot’s model for Mohr-Coulomb material that accounts for cylindrical and spherical cavities and a surcharge on the ground surface.




a)
spherical opening
of radius *a*, enclosing
unlined span
R
face
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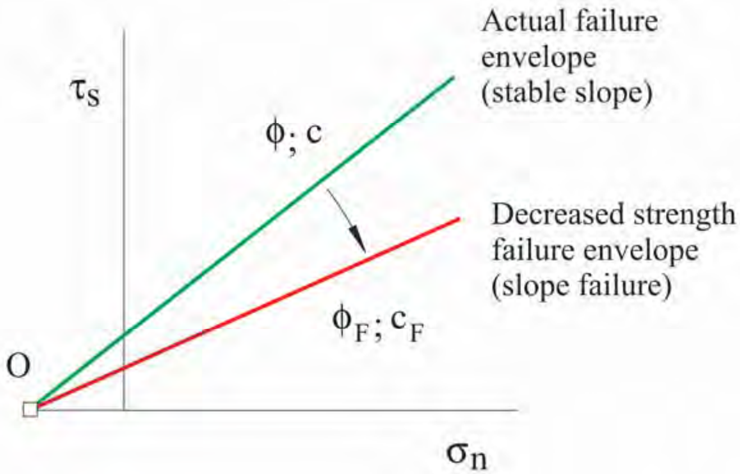
advance
direction
b)
L/2R = 0.5
support
R
face
advance
a
L
L/2R = 1.0
support
R
face
advance
a
L

Basic characteristics of the proposed model (Cont.)

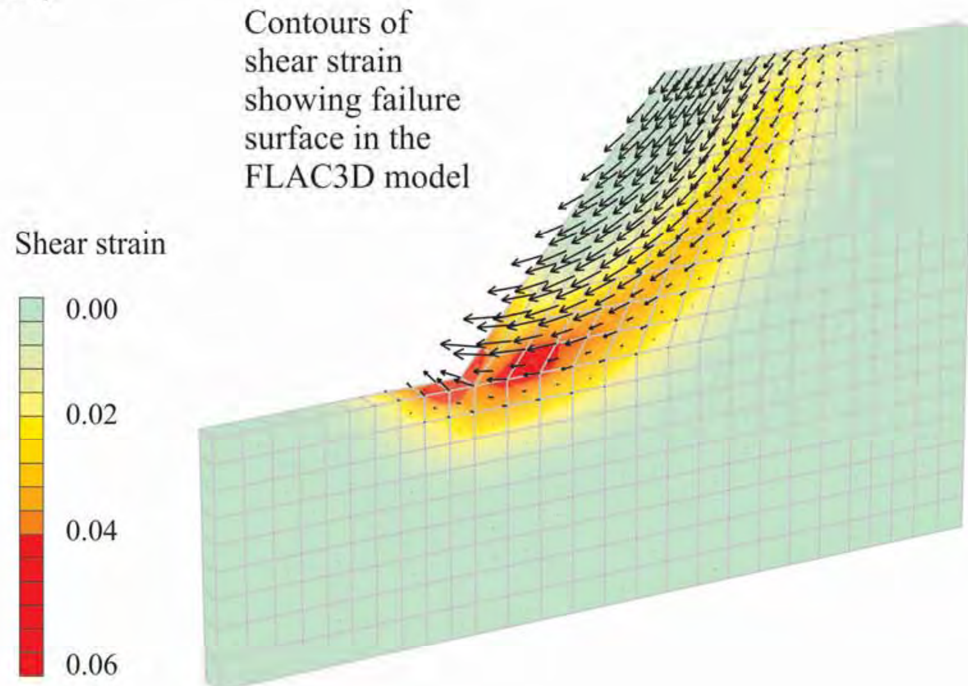
- Consideration of a Factor of Safety, FS, to quantify stability of the excavation, as commonly done with the case of slopes.

- The definition of factor of safety is agreement with the definition used in the implementation of the ‘strength reduction technique’ in commercial finite element or finite difference software.

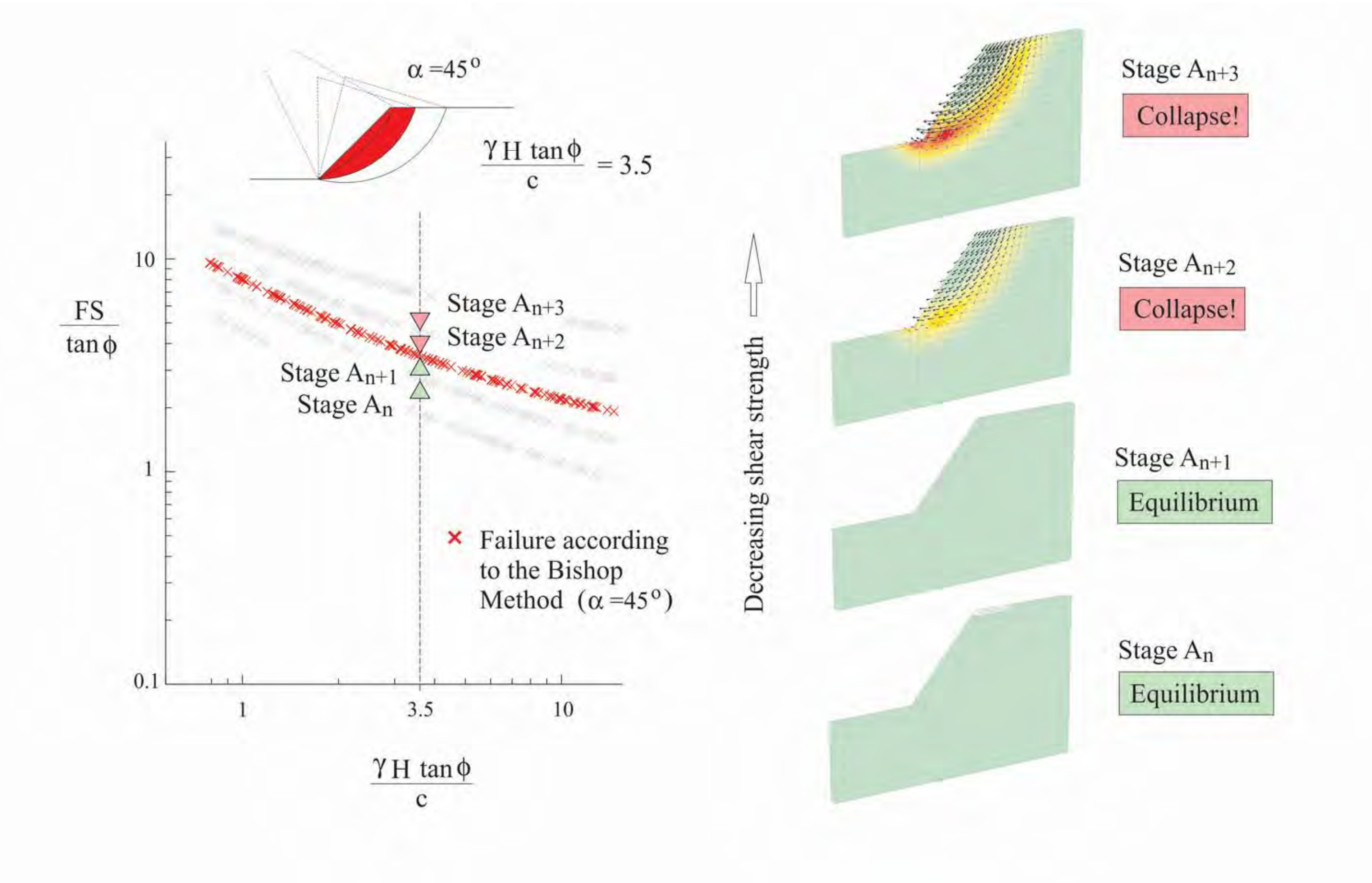
Slope stability analysis using the strength reduction technique



$$FS = \frac{c}{c_F} = \frac{\tan \phi}{\tan \phi_F}$$



Slope stability analysis using the strength reduction technique



(see equation A-14),

$$\frac{p_s}{\gamma h} = \left(\frac{q_s}{\gamma h} + 2\frac{c}{\gamma h}\frac{\sqrt{N_\phi}}{N_\phi - 1}\right)\left(\frac{h}{a}\right)^{-k(N_\phi - 1)} - \frac{1}{k(N_\phi - 1) - 1}\left[\left(\frac{h}{a}\right)^{-k(N_\phi - 1)} - \left(\frac{h}{a}\right)^{-1}\right] - 2\frac{c}{\gamma h}\frac{\sqrt{N_\phi}}{N_\phi - 1} \tag{1}$$

where the parameter k dictates the type of excavation being considered —i.e., $k = 1$ is for a cylindrical excavation and $k = 2$ is for a spherical excavation, and the parameter N_ϕ is the passive reaction coefficient defined as follows (see, for example, Terzaghi, Peck, & Mesri 1996)

$$N_\phi = \frac{1 + \sin\phi}{1 - \sin\phi} = \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \tag{2}$$

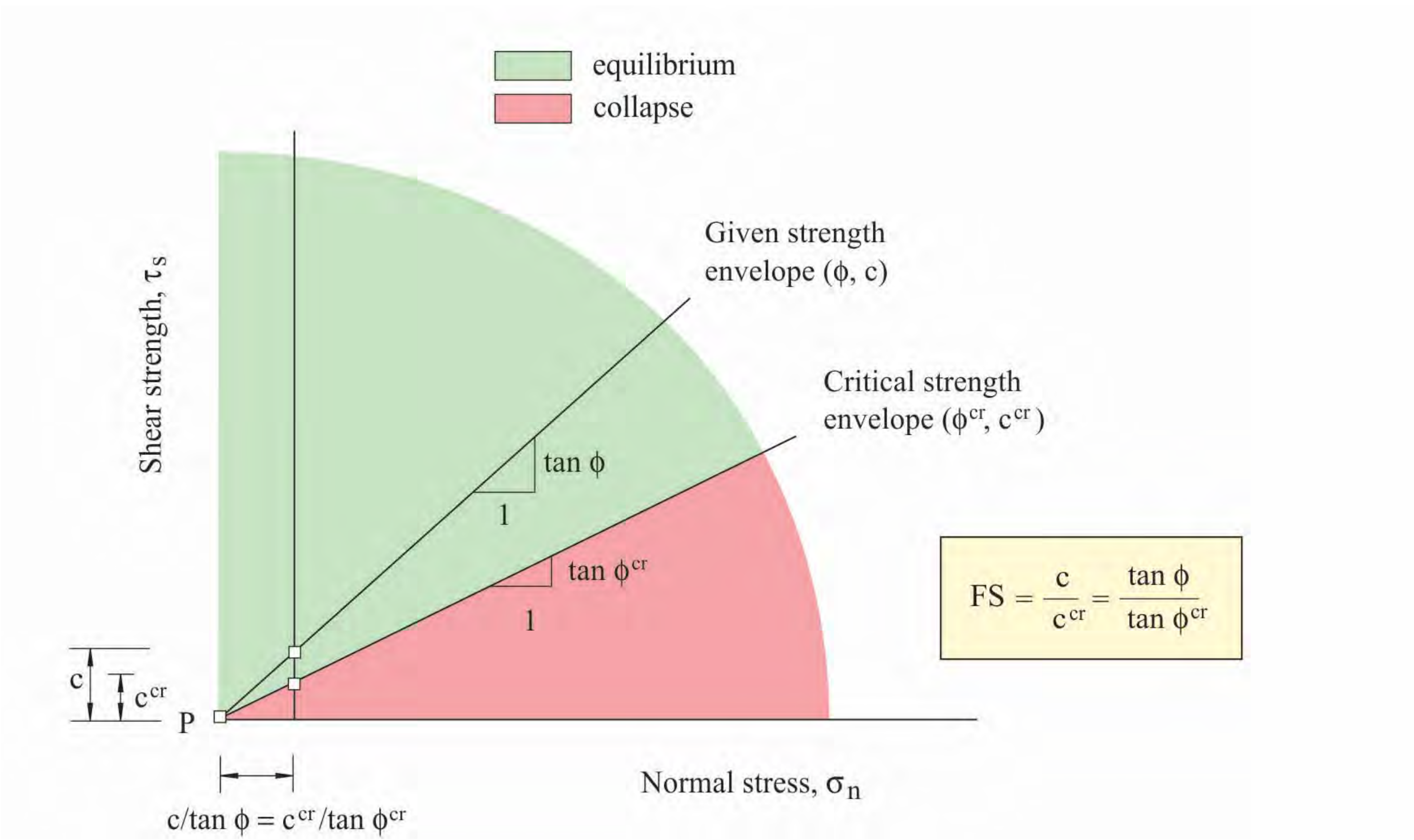
It should be noted that equation (1) is valid only when the given Mohr-Coulomb parameters for the soil lead to a state of limit equilibrium for the tunnel —the situation for which the excavation is about to collapse. In general, the strength of the material, as defined by the cohesion, c, and the internal friction angle, ϕ, will be larger than the strength associated with the critical equilibrium state of the tunnel (i.e., the parameters involved in equation 1).

Next the factor of safety, FS, for the shallow tunnel is defined to be the ratio of *actual* Mohr-Coulomb parameters and *critical* Mohr-Coulomb parameters that lead to failure of the tunnel (see Figure 10), i.e.,

$$FS = \frac{c}{c^{cr}} = \frac{\tan\phi}{\tan\phi^{cr}} \tag{3}$$

The definition of factor of safety given by equation (3) (see also Figure 10) is the very same definition of factor of safety implemented in the *strength reduction technique* for computation of factor of safety for slopes in Mohr-Coulomb materials with non-linear finite element and finite difference codes (see, for example, Zienkiewicz et al. 1975; Donald & Giam 1988; Dawson et al. 1999; Hammah et al. 2007 and 2008). In the last few years, this technique has become a standard method for computing stability conditions of surface excavations (typically slopes) and can be found

Implementation of a Factor of Safety in the extended Caquot's model



... (FS) according to Caquot's ... Coulomb material.

implemented in the most popular commercial numerical codes for analysis of excavations —for example, *FLAC* (Itasca, Inc. 2011); *Phase2* (Rocscience, Inc. 2011); *Plaxis* (Plaxis, bv 2012).

With the definition of factor of safety given by equation (3), Caquot's fundamental relationship (equation 1) can now be written as follows

$$\begin{aligned} \frac{p_s}{\gamma h} &= \left(\frac{q_s}{\gamma h} + 2\frac{c}{\gamma h}\frac{\sqrt{N_\phi}}{N_\phi - 1}\right)\left(\frac{h}{a}\right)^{-k\left(N_\phi^{FS}-1\right)} \\ &\quad - \frac{1}{k\left(N_\phi^{FS}-1\right)-1}\left[\left(\frac{h}{a}\right)^{-k\left(N_\phi^{FS}-1\right)} - \left(\frac{h}{a}\right)^{-1}\right] - 2\frac{c}{\gamma h}\frac{\sqrt{N_\phi}}{N_\phi - 1} \end{aligned} \tag{4}$$

where N_ϕ^{FS} is

$$N_\phi^{FS} = \frac{1+\sin\left(\tan^{-1}\frac{\tan\phi}{FS}\right)}{1-\sin\left(\tan^{-1}\frac{\tan\phi}{FS}\right)} \tag{5}$$

Equation (4), which is valid for any given values of Mohr-Coulomb parameters c and ϕ, allows computation of a factor of safety for the case of tunnels in frictional cohesive materials.

When the material is frictionless (i.e., $\phi = 0$ degrees and therefore $N\phi = 1$), a series of singularities appear in equation (4), which can be overcome by application of L'Hospital rule. Indeed, for frictionless materials, equation (4) becomes,

$$\frac{p_s}{\gamma h} = 1 + \frac{q_s}{\gamma h} - \left(\frac{h}{a}\right)^{-1} - 2\frac{c_{FS}}{\gamma h}k\ln\left(\frac{h}{a}\right) \tag{6}$$

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and therefore, solving for FS in equations (6) and (7), the factor of safety for the shallow tunnel can

... (FS) according to Caquot's
... Coulomb material.

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With the definition of factor of safety given by equation (3), Caquot's fundamental relationship (equation 1) can now be written as follows

$$\begin{aligned} \frac{p_s}{\gamma h} &= \left(\frac{q_s}{\gamma h} + 2 \frac{c}{\gamma h} \frac{\sqrt{N_\phi}}{N_\phi - 1} \right) \left(\frac{h}{a} \right)^{-k\left(N_\phi^{FS} - 1\right)} \\ &\quad - \frac{1}{k\left(N_\phi^{FS} - 1\right) - 1} \left[\left(\frac{h}{a} \right)^{-k\left(N_\phi^{FS} - 1\right)} - \left(\frac{h}{a} \right)^{-1} \right] - 2 \frac{c}{\gamma h} \frac{\sqrt{N_\phi}}{N_\phi - 1} \end{aligned} \tag{4}$$

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where

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and therefore, solving for FS in equations (6) and (7), the factor of safety for the shallow tunnel can

be explicitly written as follows

$$FS = \frac{2 \frac{c}{\gamma h} k \ln\left(\frac{h}{a}\right)}{1 + \frac{q_s}{\gamma h} - \frac{p_s}{\gamma h} - \left(\frac{h}{a}\right)^{-1}} \tag{8}$$

As seen from the equations above, computation of the factor of safety for the general case of frictional cohesive material requires solving the non-linear equation (4) by means of some numerical technique. Appendix B in this paper presents a computer spreadsheet and associated programming code required to compute the factor of safety, FS, from the transcendental equation (4).

5 CONSIDERATION OF WATER PORE-PRESSURE IN CAQUOT'S MODEL

When excavating shallow tunnels in soils, and as it is typically the case with other geotechnical structures like slopes and foundations, water in the ground and water on the face of the excavation itself can be expected to have an influence in the stability of the opening.

Although the *undrained* condition for the ground as it applies to saturated clays can be accounted for readily with equations (6) through (8) in Caquot's extended model, for the general case of permeable soils, as a first approximation to solving the problem, the effect of water can be accounted for by using Terzaghi's effective stress principle. This implies decomposing total stresses in the ground into effective stresses and water pressure, and computing the strength of the material in terms of effective stresses only (see, for example, Terzaghi et al. 1996). A comprehensive analysis of this type for the case of deep tunnels in permeable porous media and various hydraulic conditions for the tunnel itself (i.e., whether water pressure exists inside the tunnel or not) has been presented in Carranza-Torres & Zhao (2007). In this section, Caquot's model is further extended to consider water pressure in the ground according to Terzaghi's principle and various hydraulic conditions.

The five different cases considered here are listed in Table 1 and represented in Figures 11 through 13. The solution of these different cases can be obtained by applying a similar procedure as the one in Appendix A, this time decomposing the total (stress) problem into effective and water pressure components and applying the appropriate stress boundary conditions (which may or may not include water pressure depending on the case considered) at the crown of the tunnel and on the ground surface.

Table 1. The five hydraulic conditions considered for Caquot's

Case A. 'Dry'

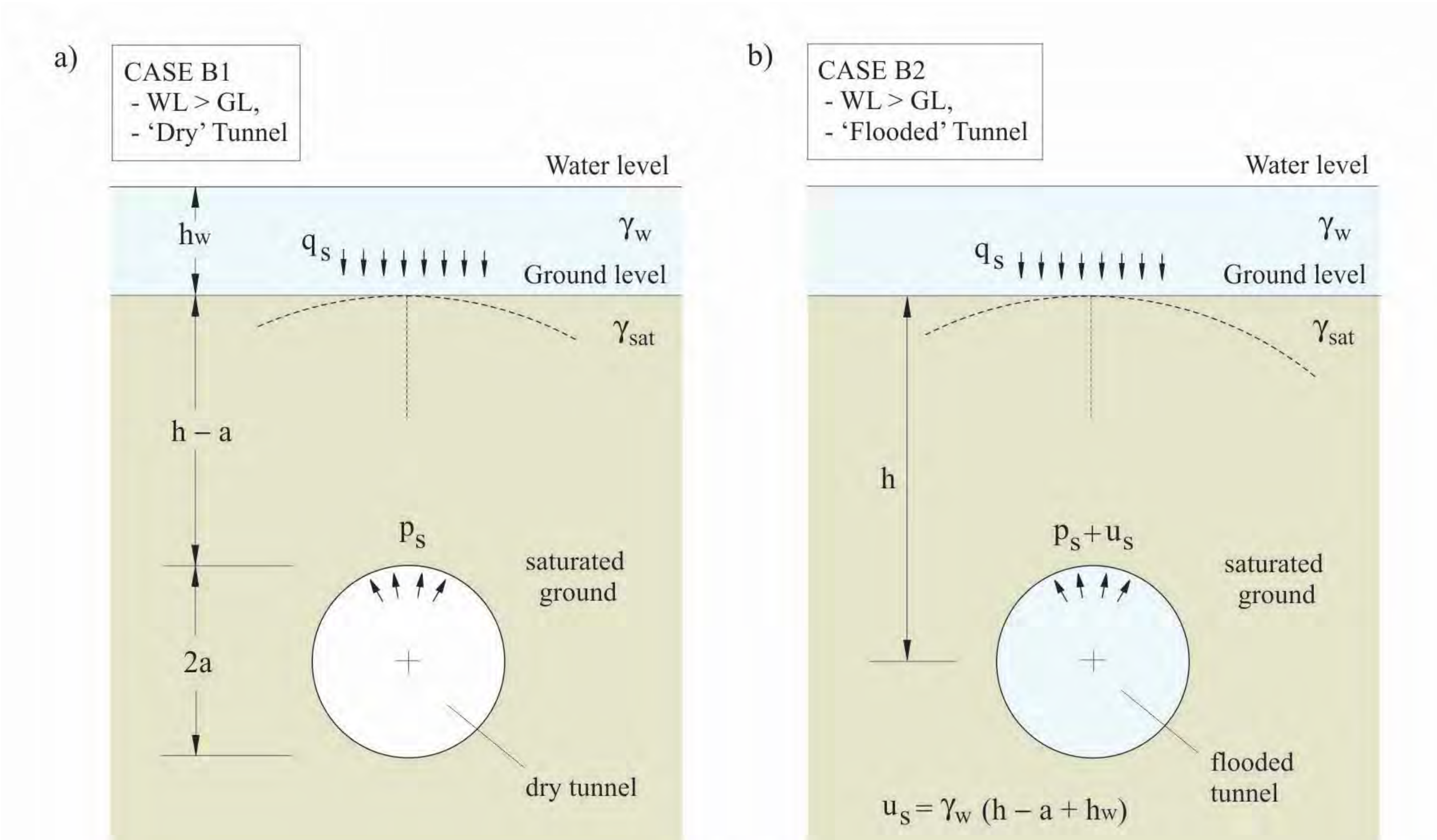
Basic characteristics of the proposed model (Cont.)

- Consideration of 'undrained' conditions or drained 'drained' conditions for water pressure in the ground, in the latter case with values of water pressure in the ground associated with a phreatic or water surface below or above the ground surface and existence of water pressure inside the tunnel (limiting cases in which tunnel is considered 'dry' or 'flooded').

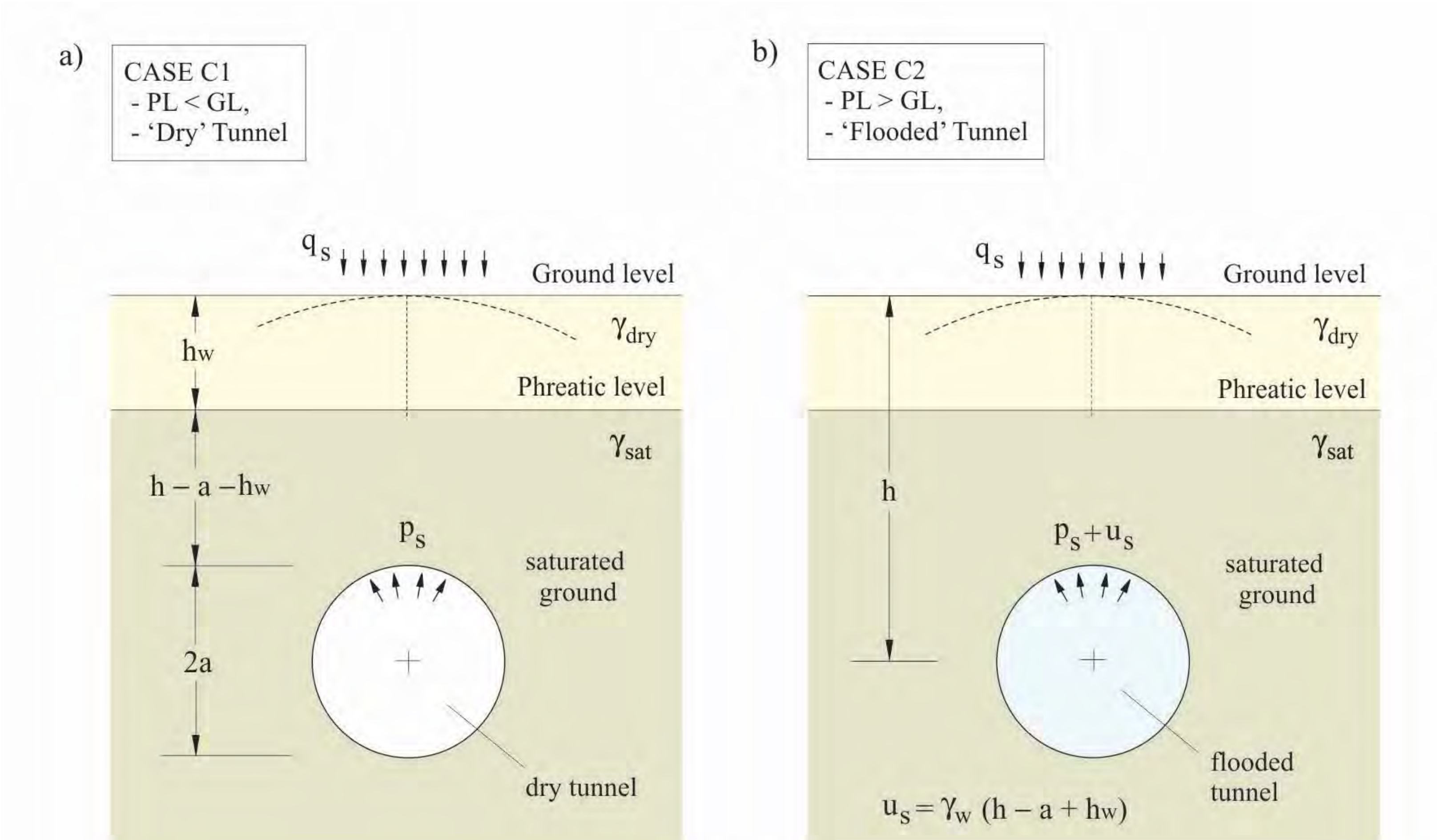
Table 1. The five hydraulic conditions considered for Caquot's extended model.

Case A.	'Dry' ground
Case B1.	'Wet' ground. Water level above ground level (WL > GL). 'Dry' tunnel
Case B2.	Same as above, but with 'flooded' tunnel
Case C1.	'Wet' ground. Phreatic level below ground level (PL < GL). 'Dry' tunnel
Case C2.	Same as Case C1, but with 'flooded' tunnel

Various hydraulic conditions considered in the extended Caquot's model



Various hydraulic conditions considered in the extended Caquot's model



Basic characteristics of the proposed model (Cont.)

- Analytical model is implemented in the form of a computer spreadsheet, implementing VBA macros.

	A	D	E
1			
2	**General Properties (for all cases)**		
3			
4	Analysis Case [1, 2]:	1	*<--1 is for cylindrical tunnels and 2 is for spherical cavities (e.g., front region)*
5	Hydraulic Conditions [1, 2, 3]:	1	*<--1 is for 'dry' conditions (no water pore pressure)*
6			*2 is is for 'wet' conditions, pore pressure defined by water level ABOVE ground level*
7			*3 is for 'wet' conditions, pore pressure defined by a phreatic surface (water level BELOW ground surface)*
8	Cylinder or sphere Radius [m]:	1	
9	Depth Tunnel Axis [m]:	3	
10			
11	Dry unit weight [kN/m3]:	18	
12			
13	Mohr-Coulomb friction [deg]:	30	*<-- always 'effective' material properties*
14	Mohr-Coulomb cohesion [kPa]:	9	*<-- always 'effective' material properties*
15			
16	Ground surface surcharge [kPa]:	72	
17	Tunnel support pressure [kPa]:	36	
18			
19	**Additional Properties for 'Wet' Conditions (cases 2 and 3 above)**		
20			
21	Water pressure case [1, 2]:	1	*<-- 1 is for dry excavation (pumping out of water flowing into tunnel)*
22			*<-- 2 is for flooded tunnel (excavation in water, without disturbance to surrounding hydrostatic pore pressure)*
23	Distance between water surface		
24	and ground surface [m]:	1.5	*<-- always positive value*
25			
26	Saturated unit weight [kN/m3]:	22	
27	Unit weight water [kN/m3]:	10	
28			Compute Results / Clear Results
29	**Results**		
30			
31	Factor of Safety Coefficient:	1.71	
32			
33	Comments on results	Factor of Safety computed successfully...	
34			

Structure of this presentation

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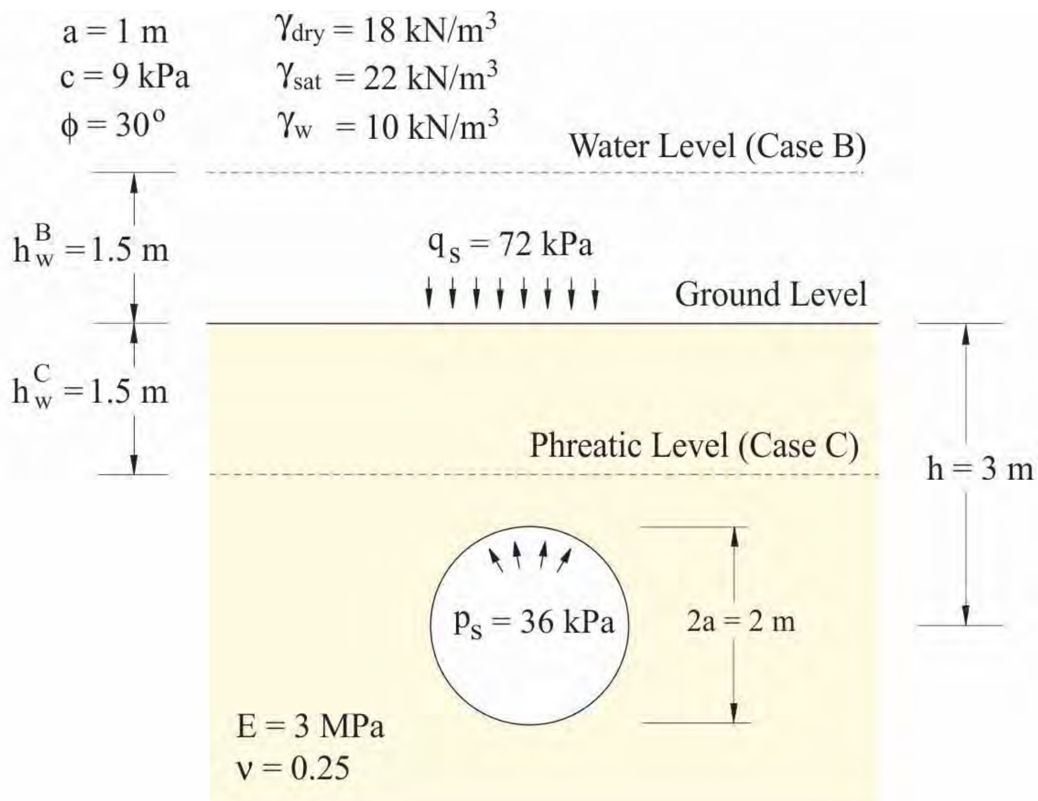
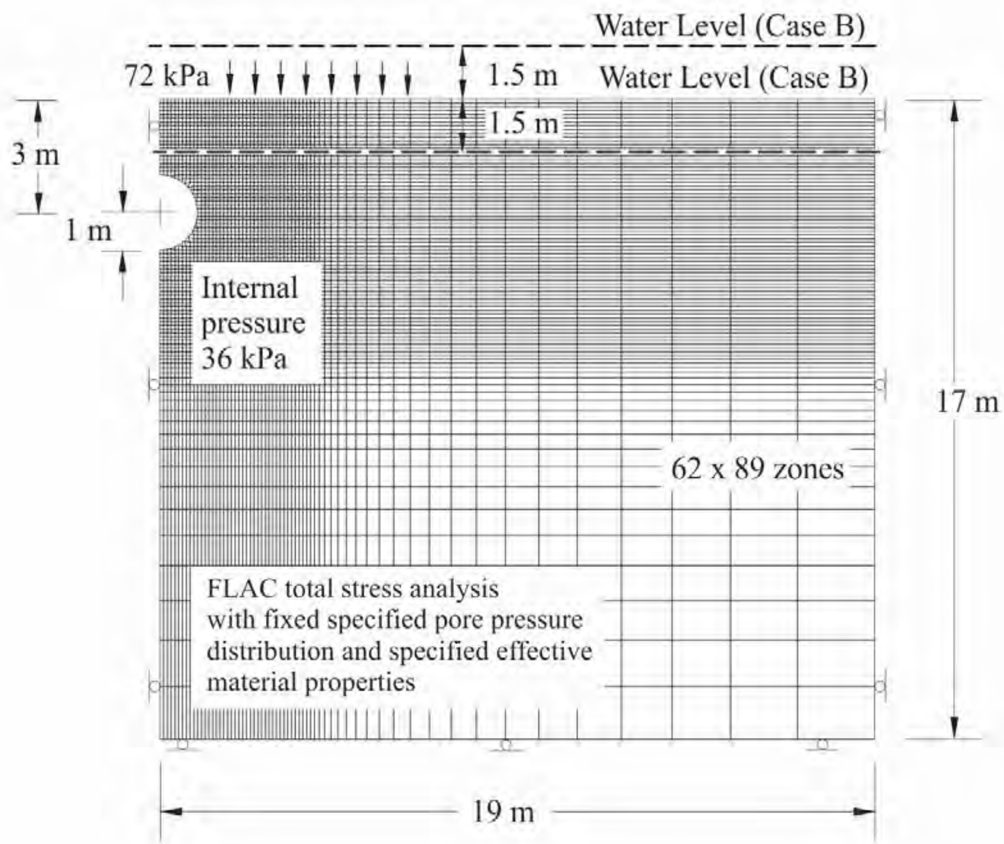
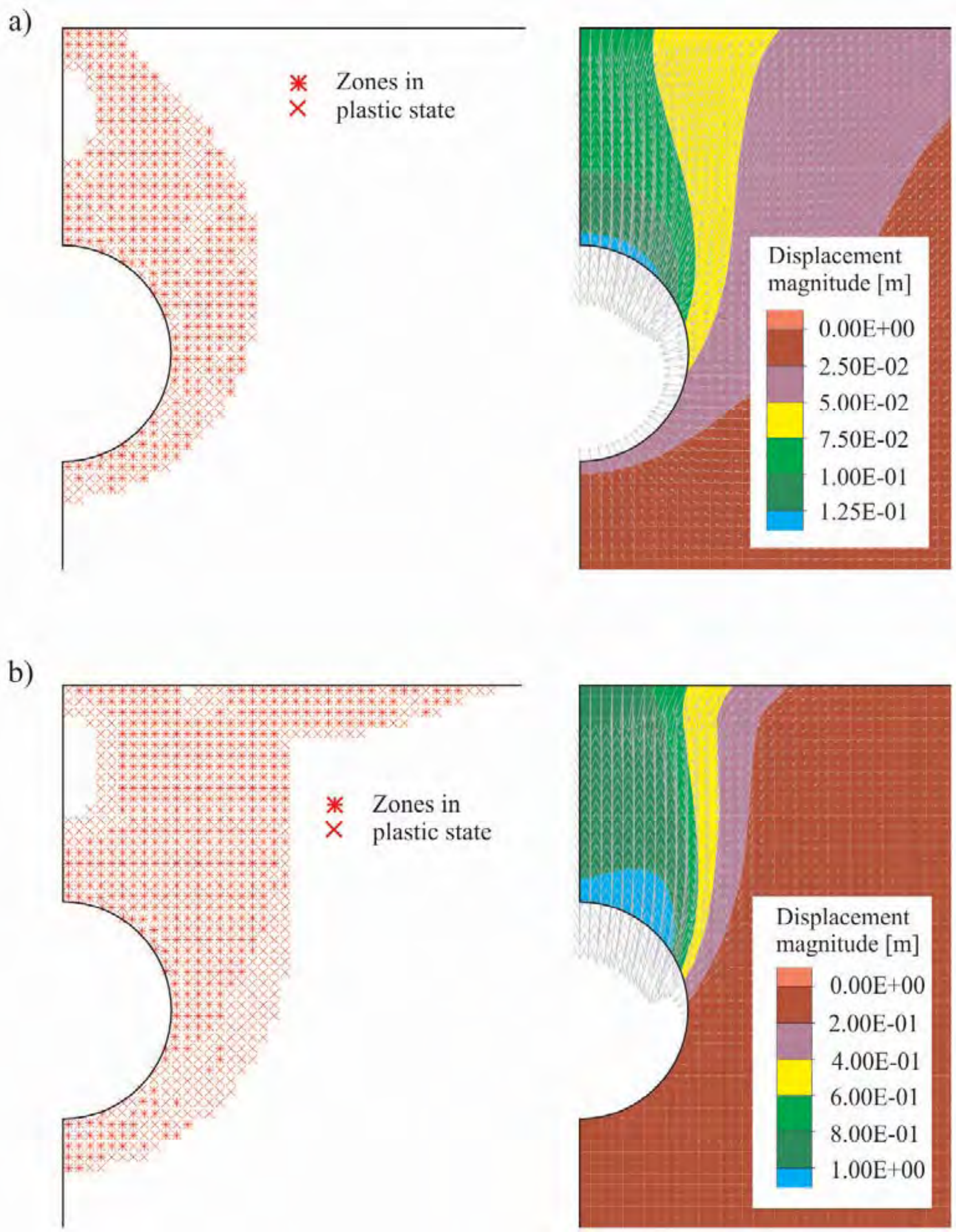
- Final comments.

Comparison of factor of safety results obtained with analytical and numerical models



Comparison of factor of safety results obtained with analytical and numerical models (Cont.)



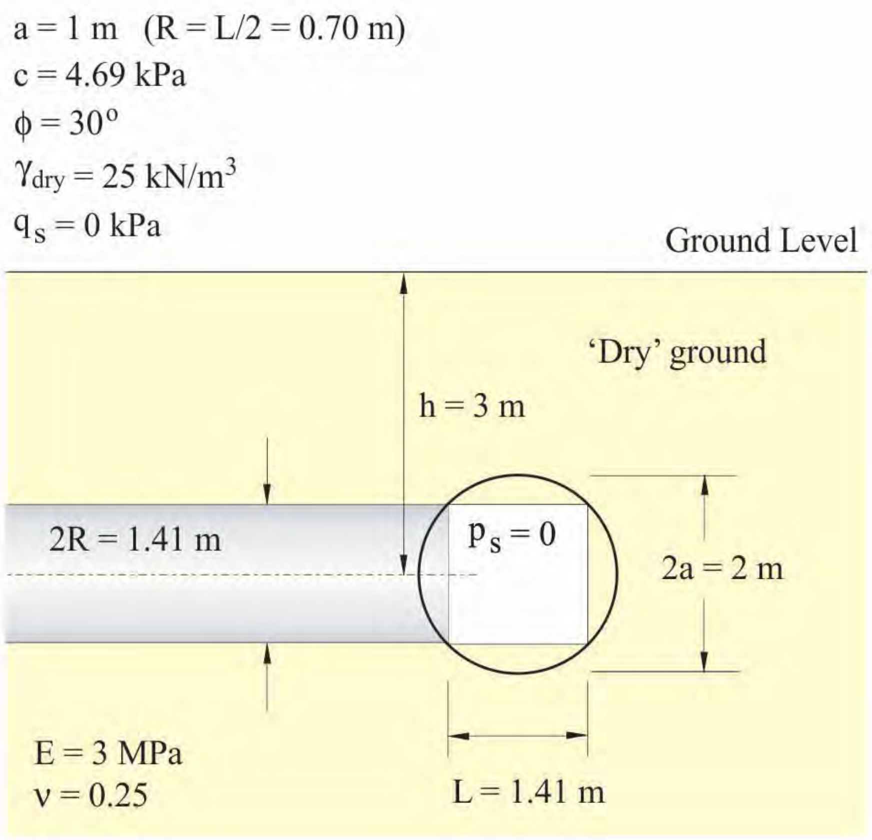

a)
Zones in
plastic state
Displacement
magnitude [m]
0.00E+00
2.50E-02
5.00E-02
7.50E-02
1.00E-01
1.25E-01
b)
Zones in
plastic state
Displacement
magnitude [m]
0.00E+00
2.00E-01
4.00E-01
6.00E-01
8.00E-01
1.00E+00

Comparison of factor of safety results obtained with analytical and numerical models

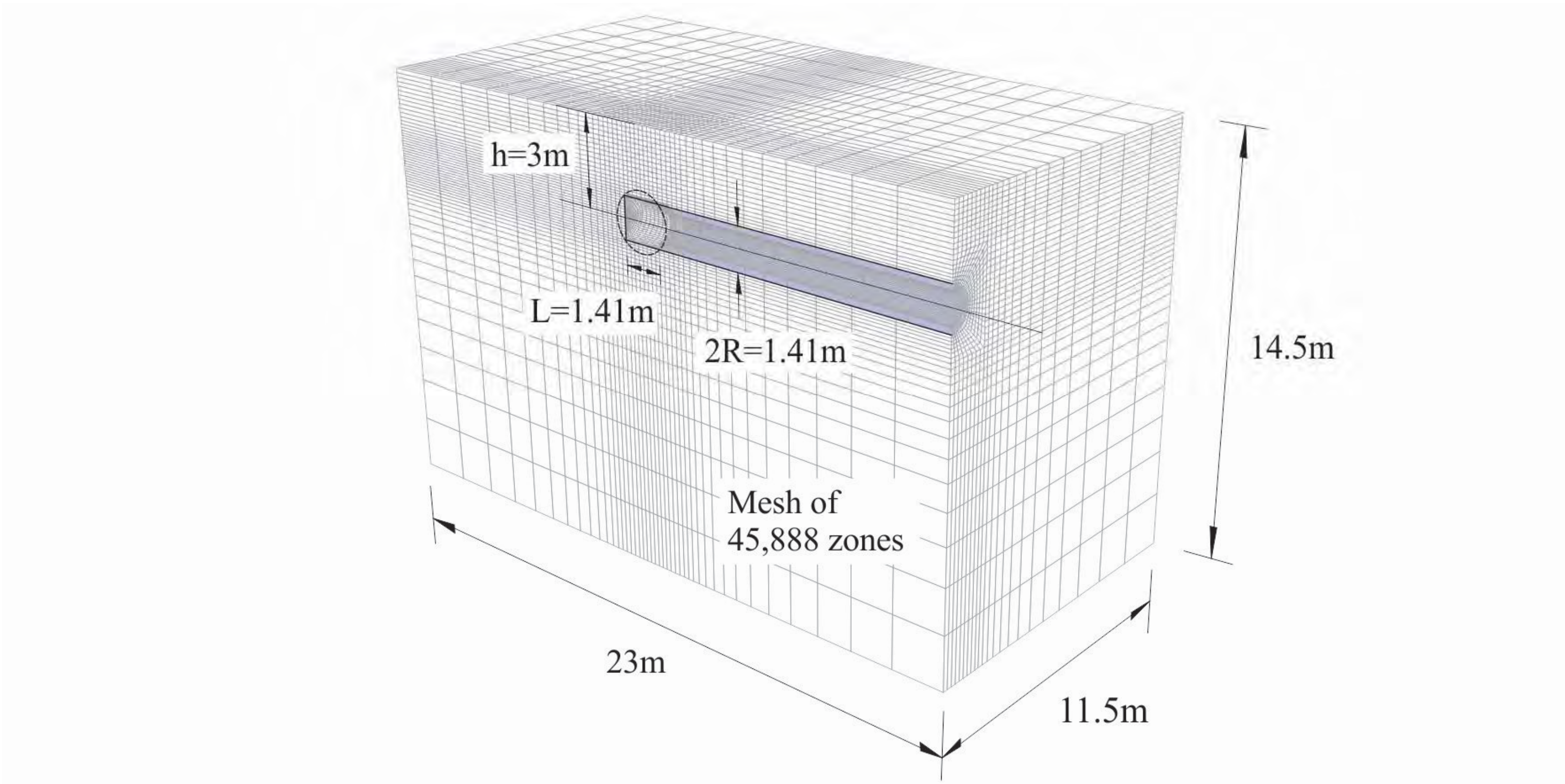
Table 2. Factor of safety results for the problem in Figure 14.

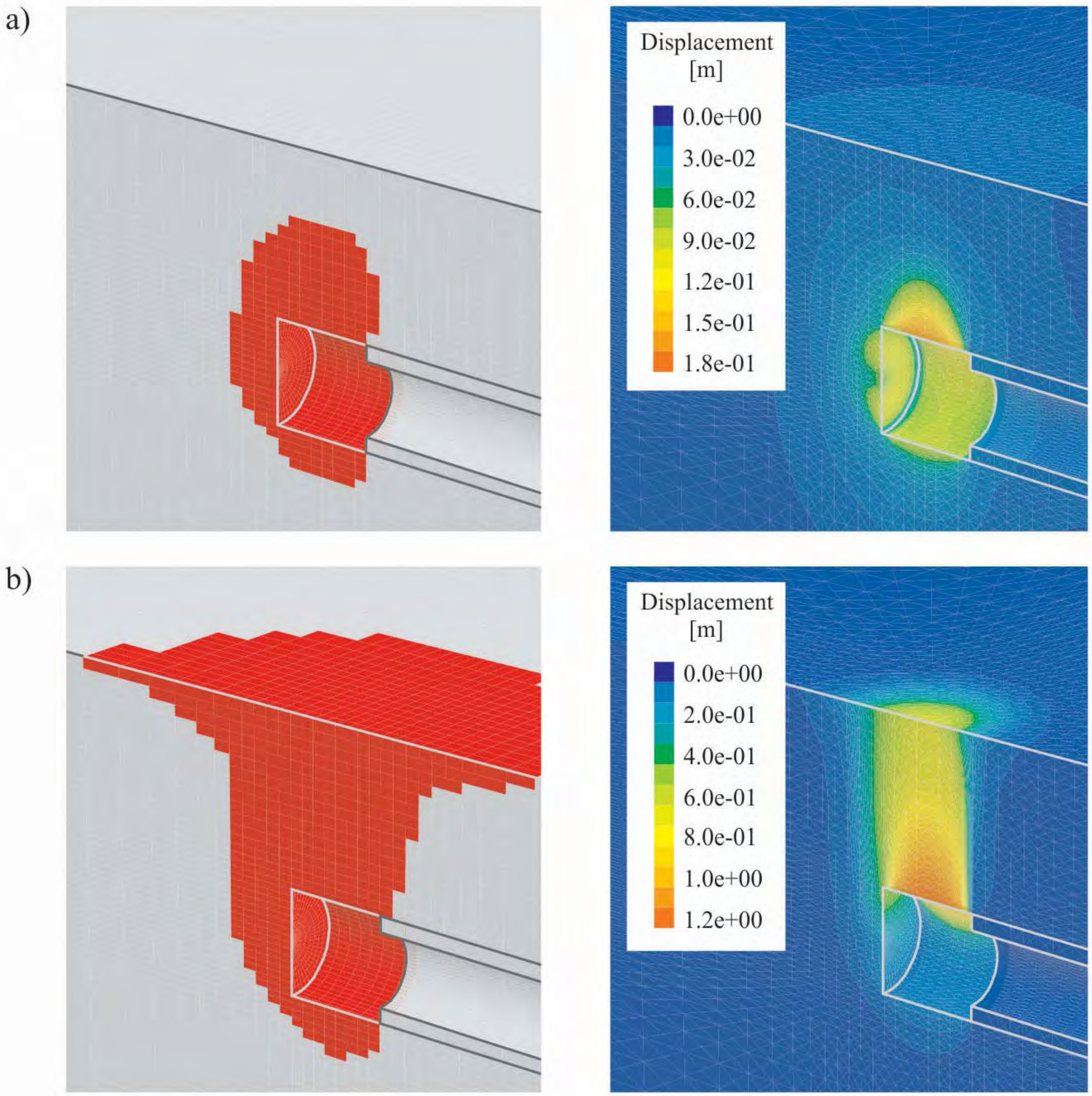
Case Type	Caquot's extended (Spreadsheet)	Numerical (*FLAC*)
A- 'Dry' ground	1.71	2.01
B1- WL > GL; 'dry' tunnel	0.97	1.03
B2- WL > GL; 'flooded' tunnel	1.96	2.25
C1- PL < GL; 'dry' tunnel	1.61	1.74
C2- PL < GL; 'flooded' tunnel	1.76	2.02

Comparison of factor of safety results obtained with analytical and numerical models (Cont.)



Comparison of factor of safety results obtained with analytical and numerical models (Cont.)





a)
Displacement
[m]
0.0e+00
3.0e-02
6.0e-02
9.0e-02
1.2e-01
1.5e-01
1.8e-01
b)
Displacement
[m]
0.0e+00
2.0e-01
4.0e-01
6.0e-01
8.0e-01
1.0e+00
1.2e+00

Comparison of factor of safety results obtained with analytical and numerical models

Table 3. Factor of safety results for the problem in Figure 18.

Type of solution	Factor of safety
Caquot's extended solution (spreadsheet)	1.00
Numerical, *FLAC*^{3D}	1.10
Limit equilibrium solution (model in Figure 3b	0.75

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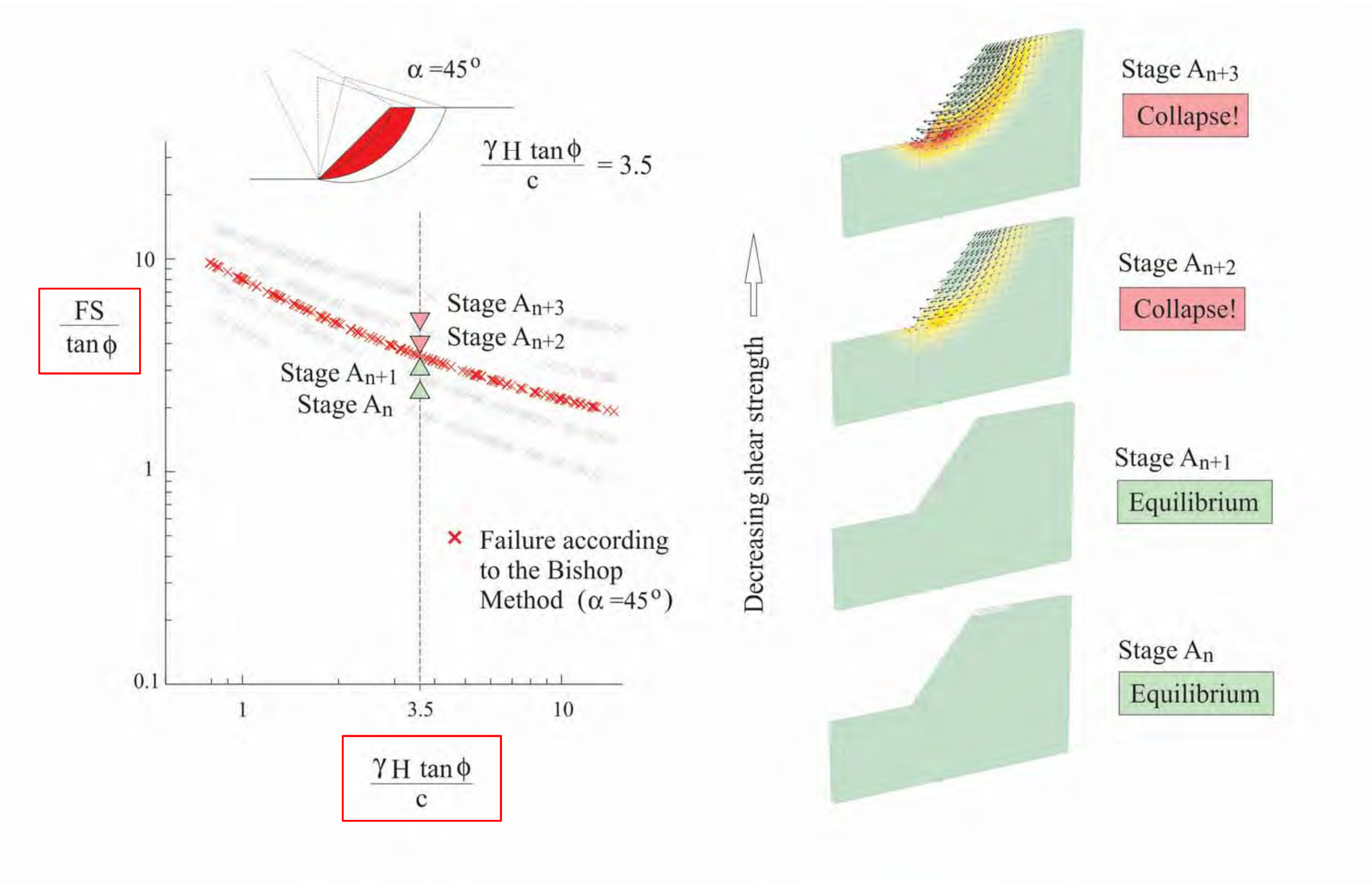
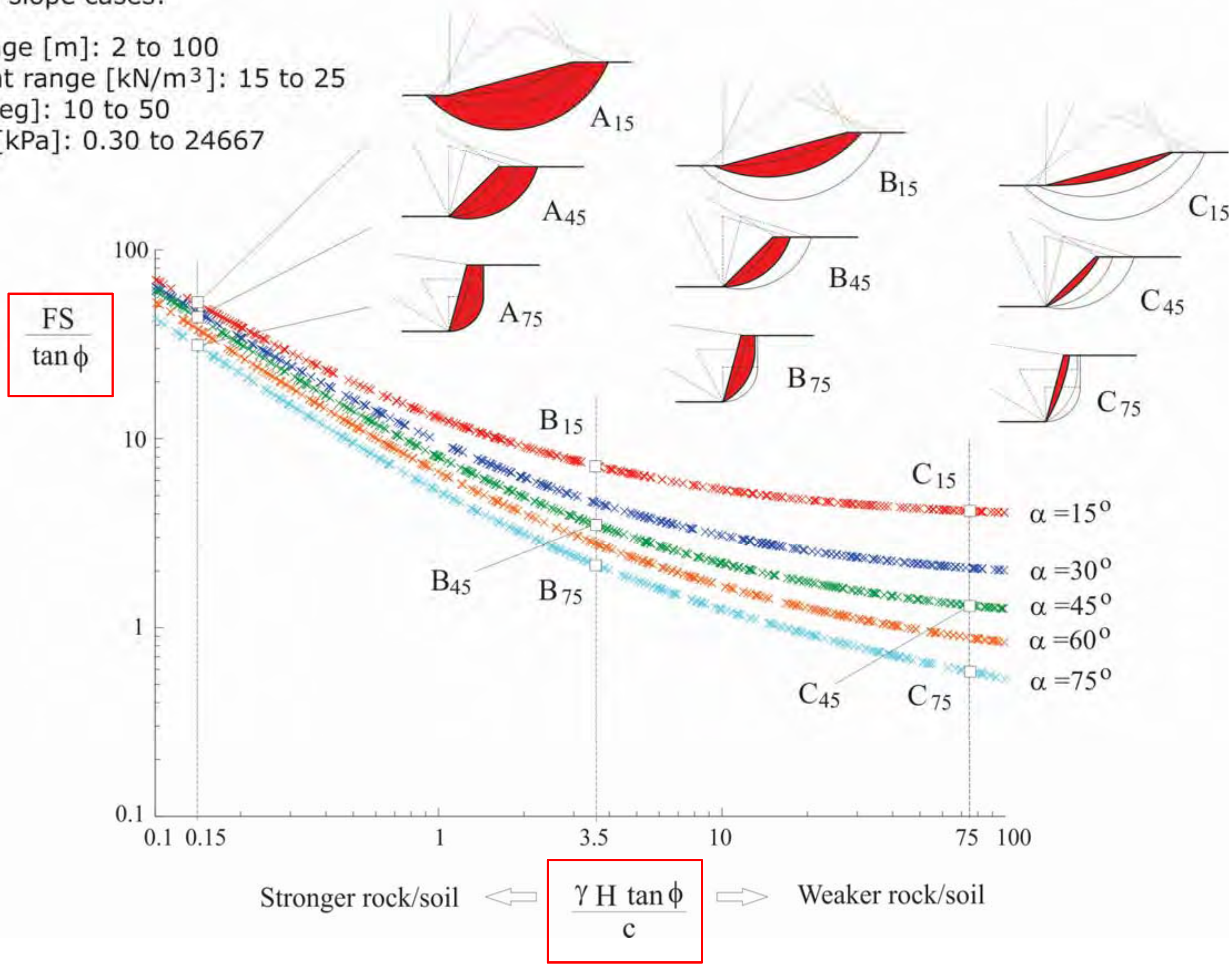
Slope stability analysis using the strength reduction technique



Scaling of factor of safety for slopes excavated in Mohr-Coulomb materials

Results from 1500 randomly generated slope cases:

Height range [m]: 2 to 100
Unit weight range [kN/m³]: 15 to 25
Friction [deg]: 10 to 50
Cohesion [kPa]: 0.30 to 24667



From Hoek and Bray (1981) Rock Slope Engineering. Institute of Mining and Metallurgy, London.

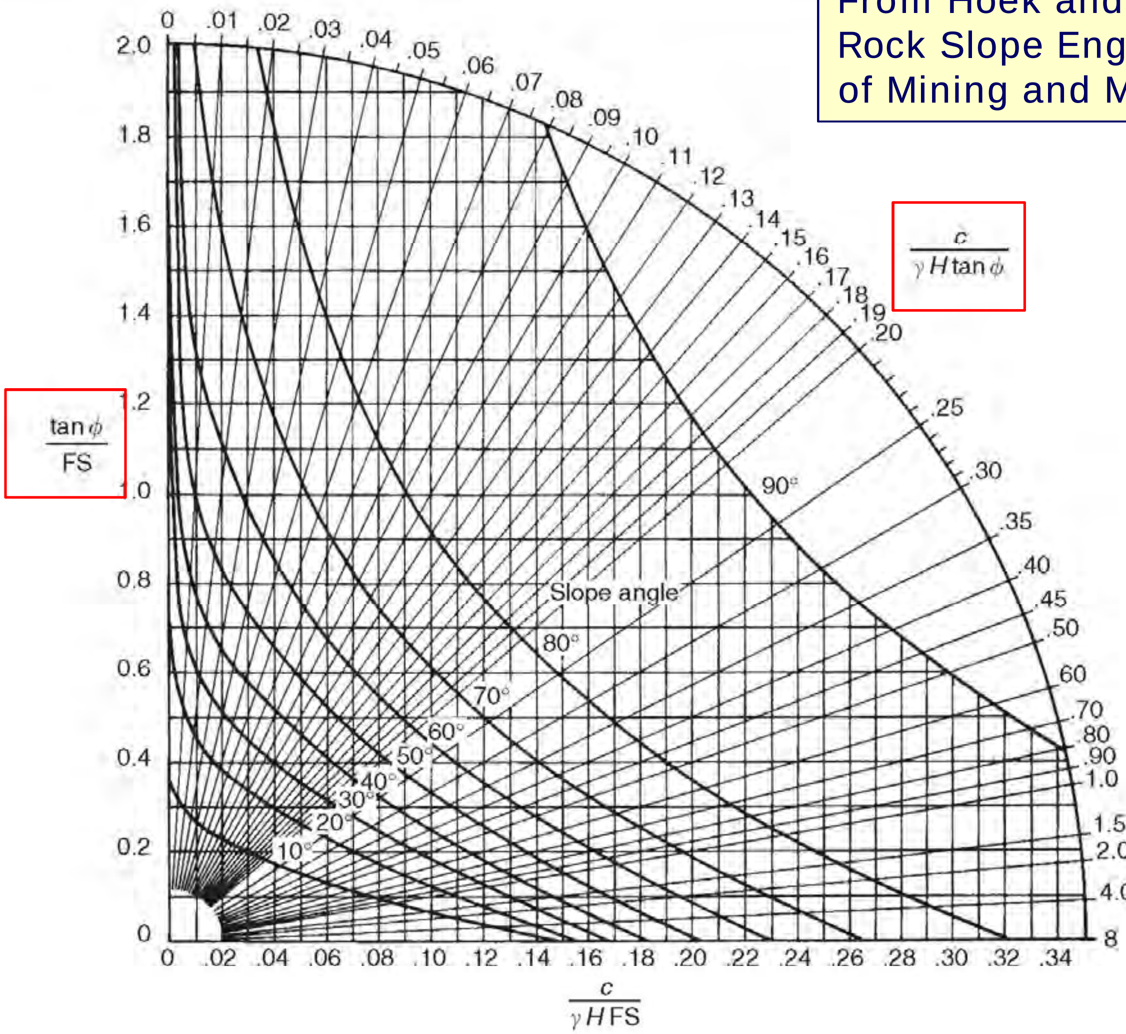


Figure 8.7 Circular failure chart number 2—ground water condition 2 (Figure 8.5).

Because of the

according to Caquot's

material.

implemented in the most popular commercial numerical codes for analysis of excavations —for example, *FLAC* (Itasca, Inc. 2011); *Phase2* (Rocscience, Inc. 2011); *Plaxis* (Plaxis, bv 2012).

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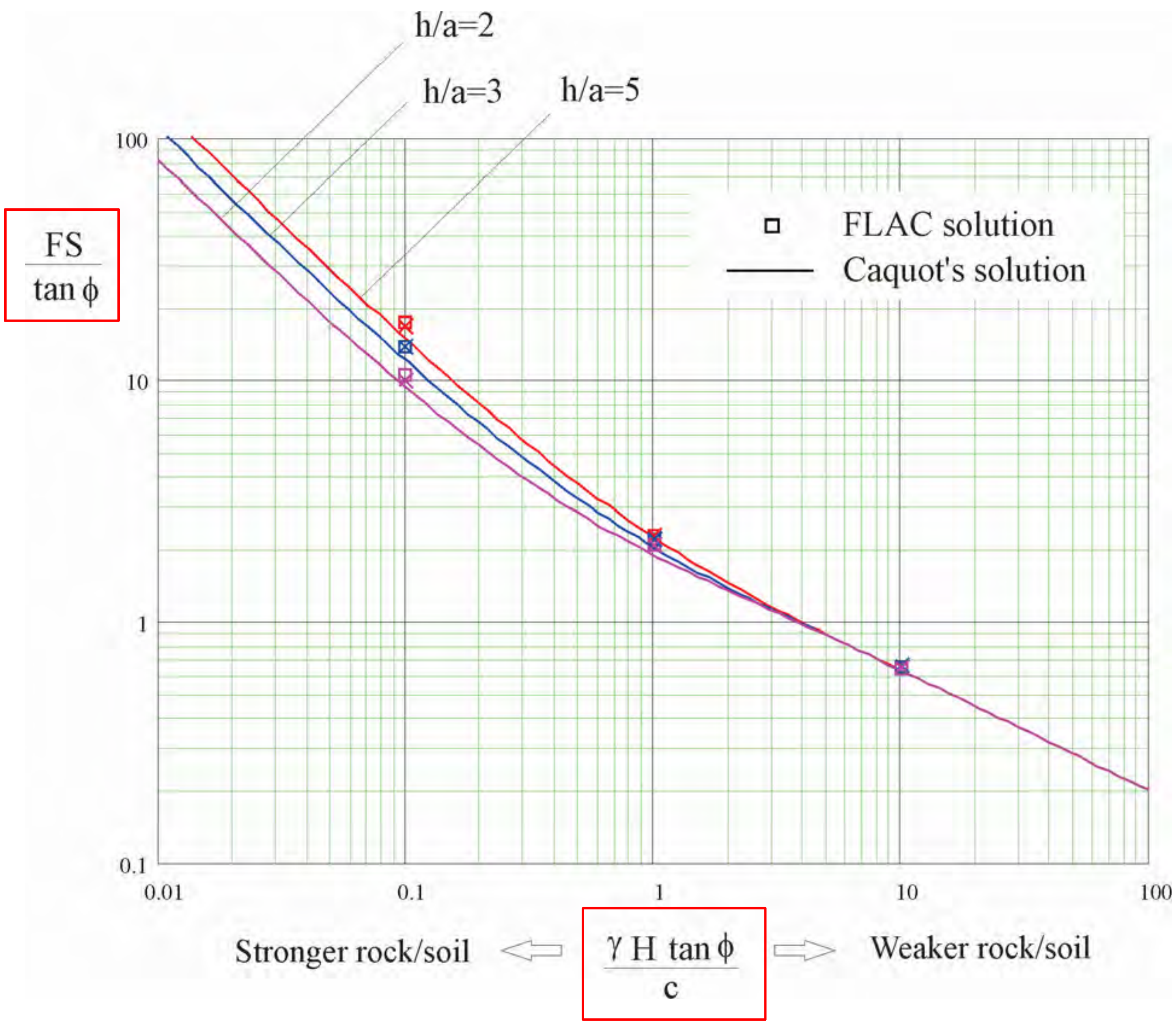
Scaling of factor of safety for shallow excavated in Mohr-Coulomb materials (cylindrical cavity and zero internal pressure)



(FS) according to Caquot's
Coulomb material.

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and therefore, solving for *FS* in equations (6) and (7), the factor of safety for the shallow tunnel can be explicitly written as follows

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As seen from the equations above, computation of the factor of safety for the general case of frictional cohesive material requires solving the non-linear equation (4) by means of some numerical technique. Appendix B in this paper presents a computer spreadsheet and associated programming code required to compute the factor of safety, FS, from the transcendental equation (4).

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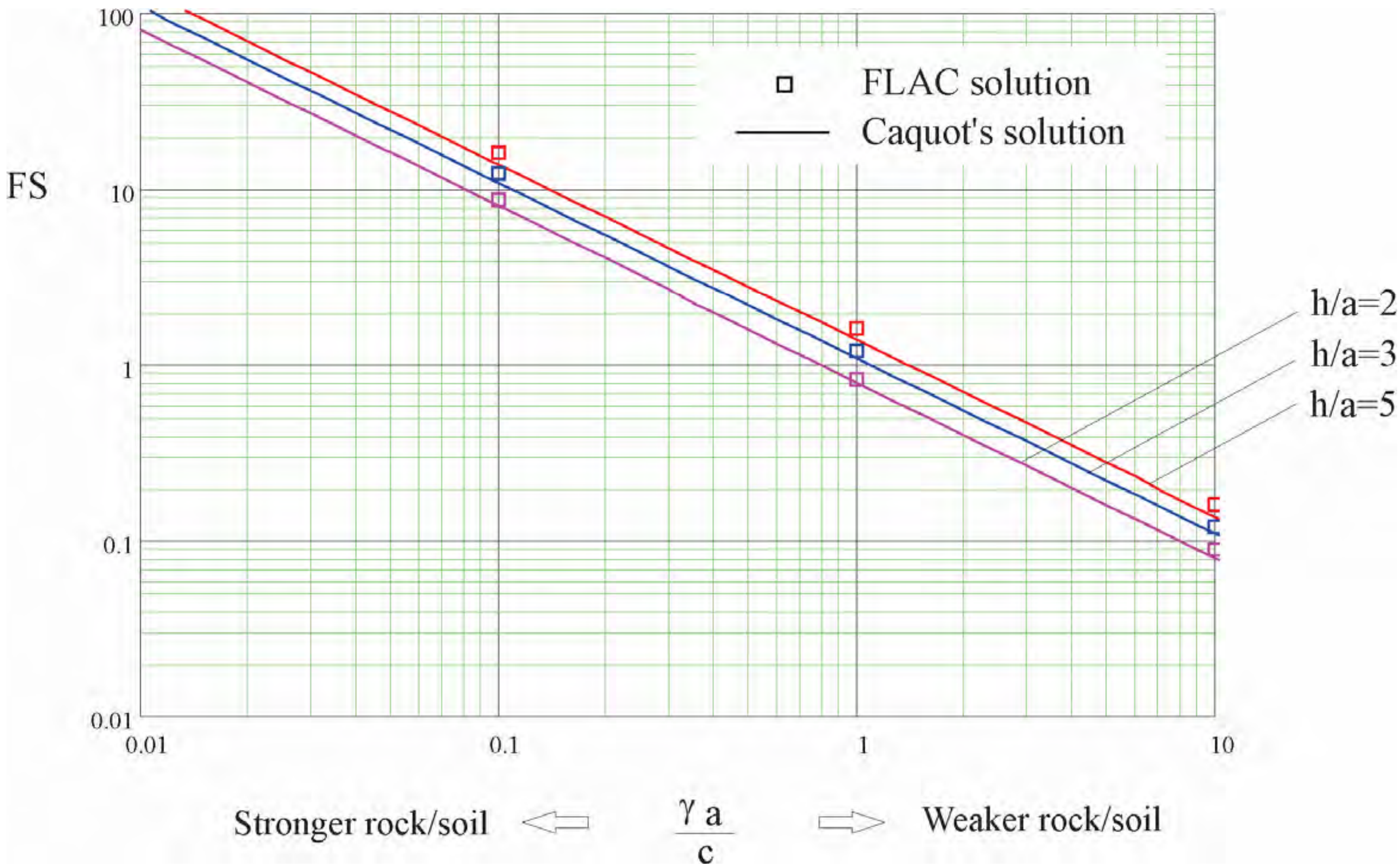
Table 1. The five hydraulic conditions considered for Caquot's

Case A ‘Dry’

Scaling of factor of safety for shallow excavated in *cohesionless* Mohr-Coulomb materials, or Tresca materials (cylindrical cavity and zero internal pressure)



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Final comments

- Introduction of a factor of safety to assess stability for shallow tunnels, as commonly done in the case of slopes, could be an useful indicator of how far or close a designed (section or front of) tunnel is from collapse, particularly at the early stages of design, when different alternatives for support are considered (or alternatively, if remedial works dictate removing support, as in the case of the Hollywood Boulevard tunnel collapse described earlier on).

- With the implementation of the ‘strength reduction technique’ to compute factor of safety in most commercial software for analysis of geotechnical problems (FLAC, Phase2, Plaxis and others), there is an opportunity to explore the application of the concept of factor of safety for shallow tunnels (as has been traditionally done with the case of slopes) –indeed, one of the objectives of our paper is to revive discussions on the topic.

Final comments (Cont.)

- Despite the availability of the strength reduction technique in most software for numerical analysis of geotechnical problems, an analytical solution that allows fast assessment of the stability conditions of shallow tunnels (e.g., through a factor of safety) will allow implementation of statistical techniques (such as Monte Carlo simulations) to account for variability and uncertainty in ground variables, and so compute probability of failure and reliability of the design (as currently done for the case of slopes and other surface excavations).

Final comments (Cont.)

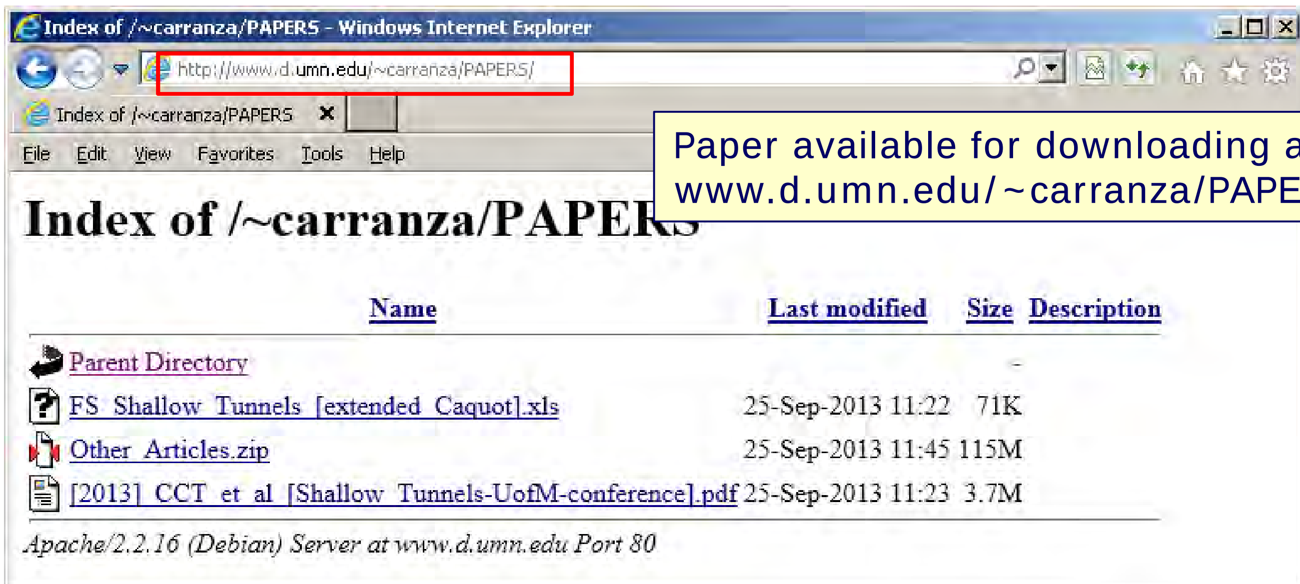
- The analysis presented in the paper is by no means complete and further developments are possible. Two of these are listed below (others are listed in the paper.

- Factor of safety results obtained with the approximate Caquot's solution and with the strength reduction technique in finite element and finite difference models, need to be summarized in dimensionless representations, i.e., charts equivalent to slope stability charts, from where regression analysis could be intended (e.g., to provide equations to compute factor of safety of shallow tunnels in terms of dimensionless variables).

- The limitations of Caquot's solution have to be evaluated further, in particular, in regard to stresses in the ground prior to excavation (e.g., lateral earth pressure coefficient) and distribution of pressure inside the tunnel. This can be achieved by comparison of Caquot's models and those obtained with the strength reduction technique in numerical models.

Stability of shallow circular tunnels in soils using analytical and numerical models

C. Carranza-Torres, T. Reich & D. Saftner
Department of Civil Engineering, University of Minnesota, Duluth Campus, Minnesota, USA



Paper available for downloading at www.d.umn.edu/~carranza/PAPERS/

1 INTRODUCTION

The stability of shallow circular cavities excavated in soils is
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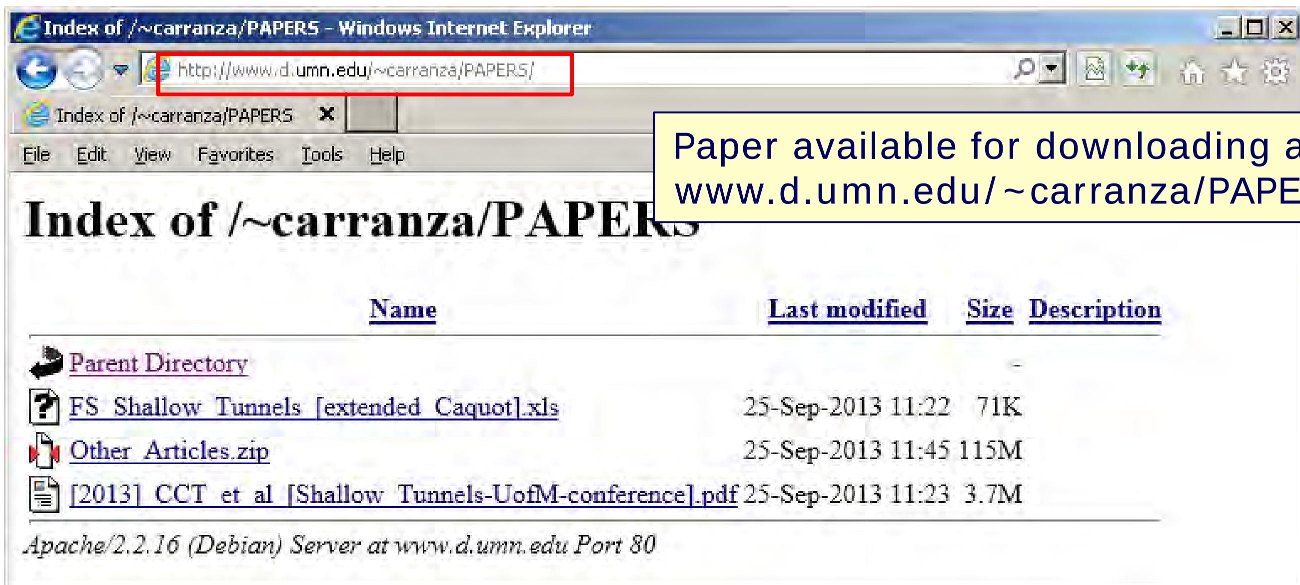
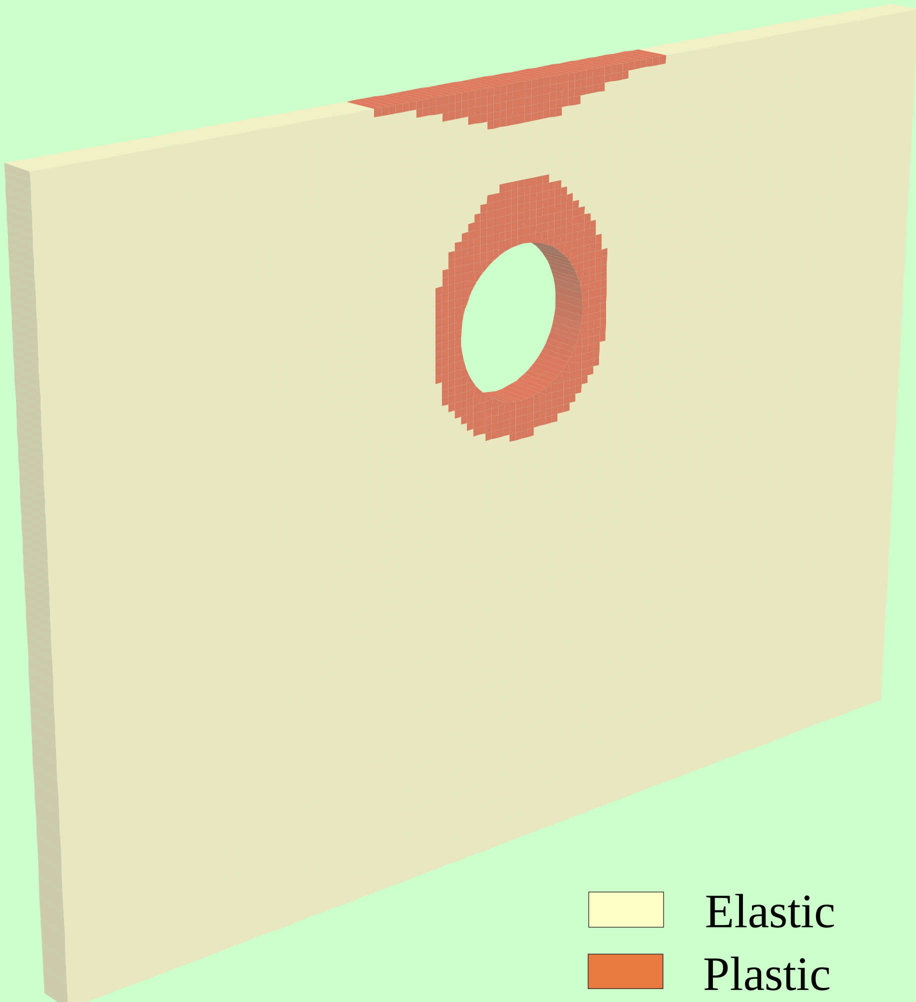
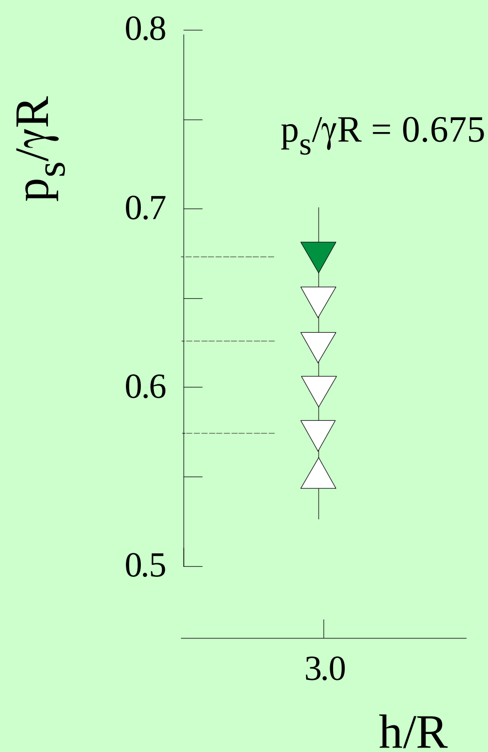


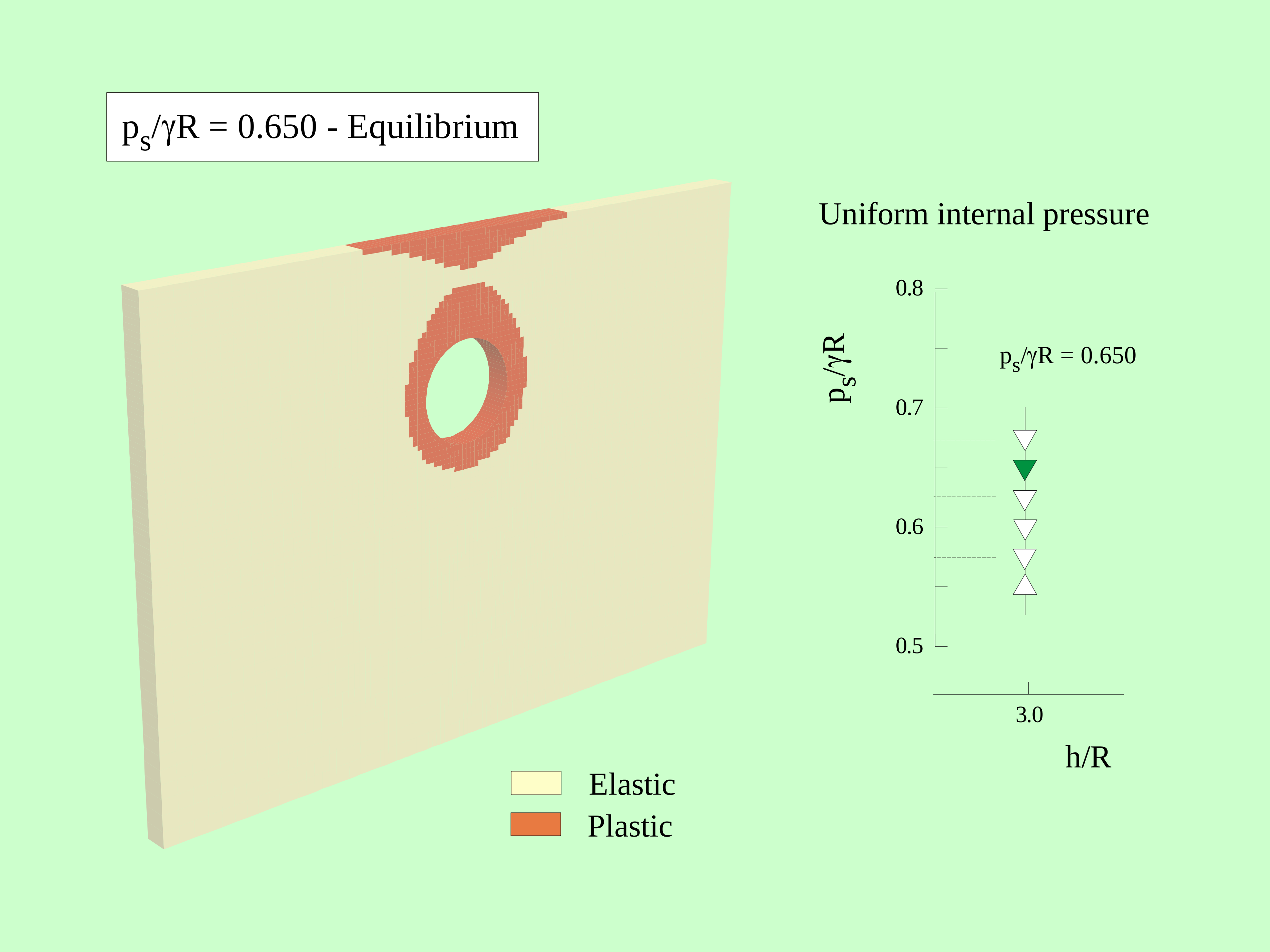
Paper available for downloading at
www.d.umn.edu/~carranza/PAPERS/

$p_s/\gamma R = 0.675$ - Equilibrium



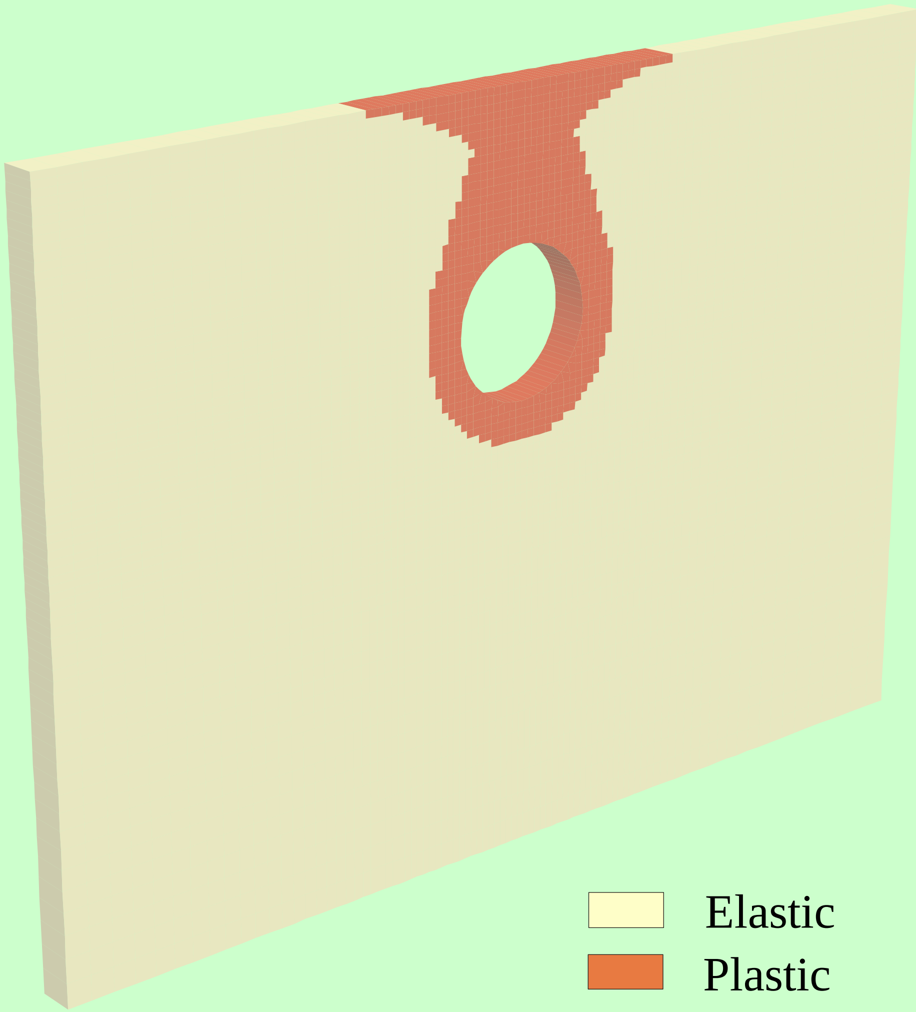
Uniform internal pressure



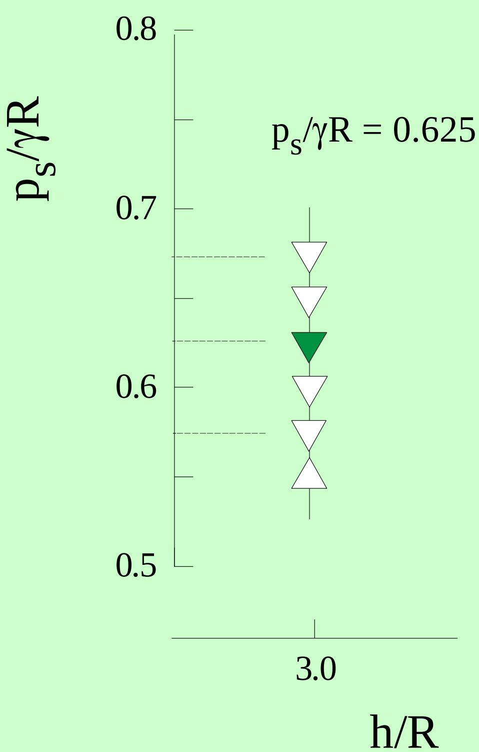


$p_s/\gamma R$ = 0.650 - Equilibrium
Uniform internal pressure
0.8
0.7
0.6
0.5
$p_s/\gamma R$
$p_s/\gamma R$ = 0.650
3.0
h/R
Elastic
Plastic

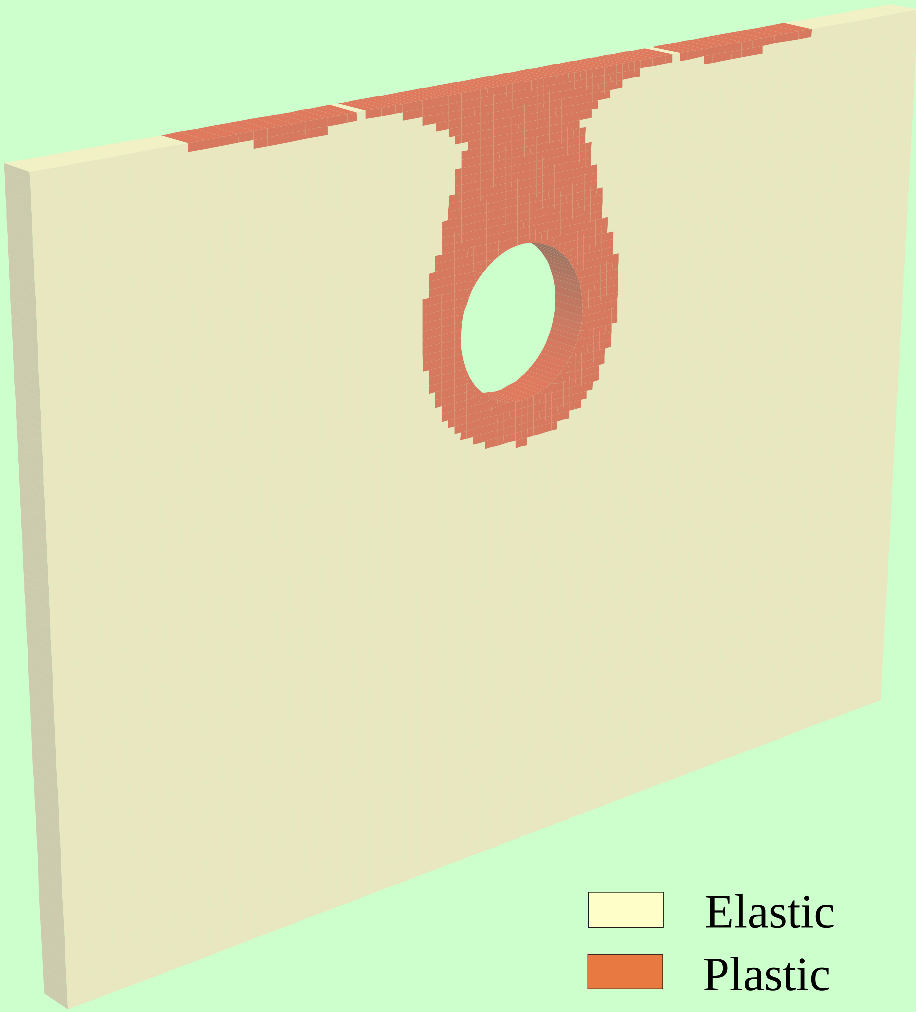
$p_s/\gamma R = 0.625$ - Equilibrium



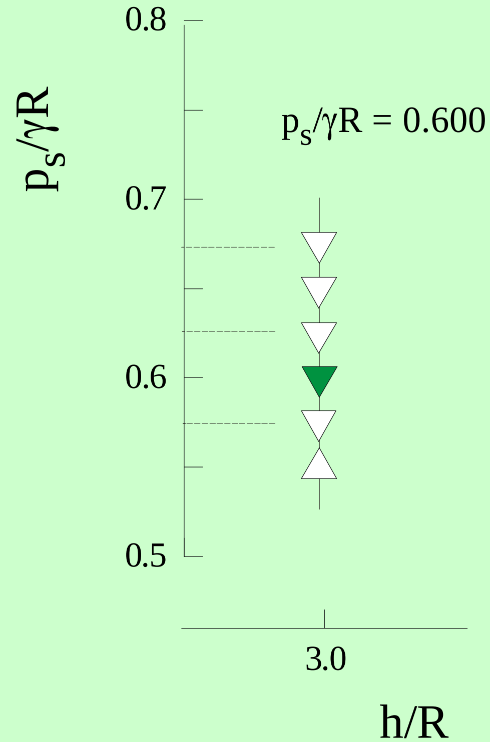
Uniform internal pressure



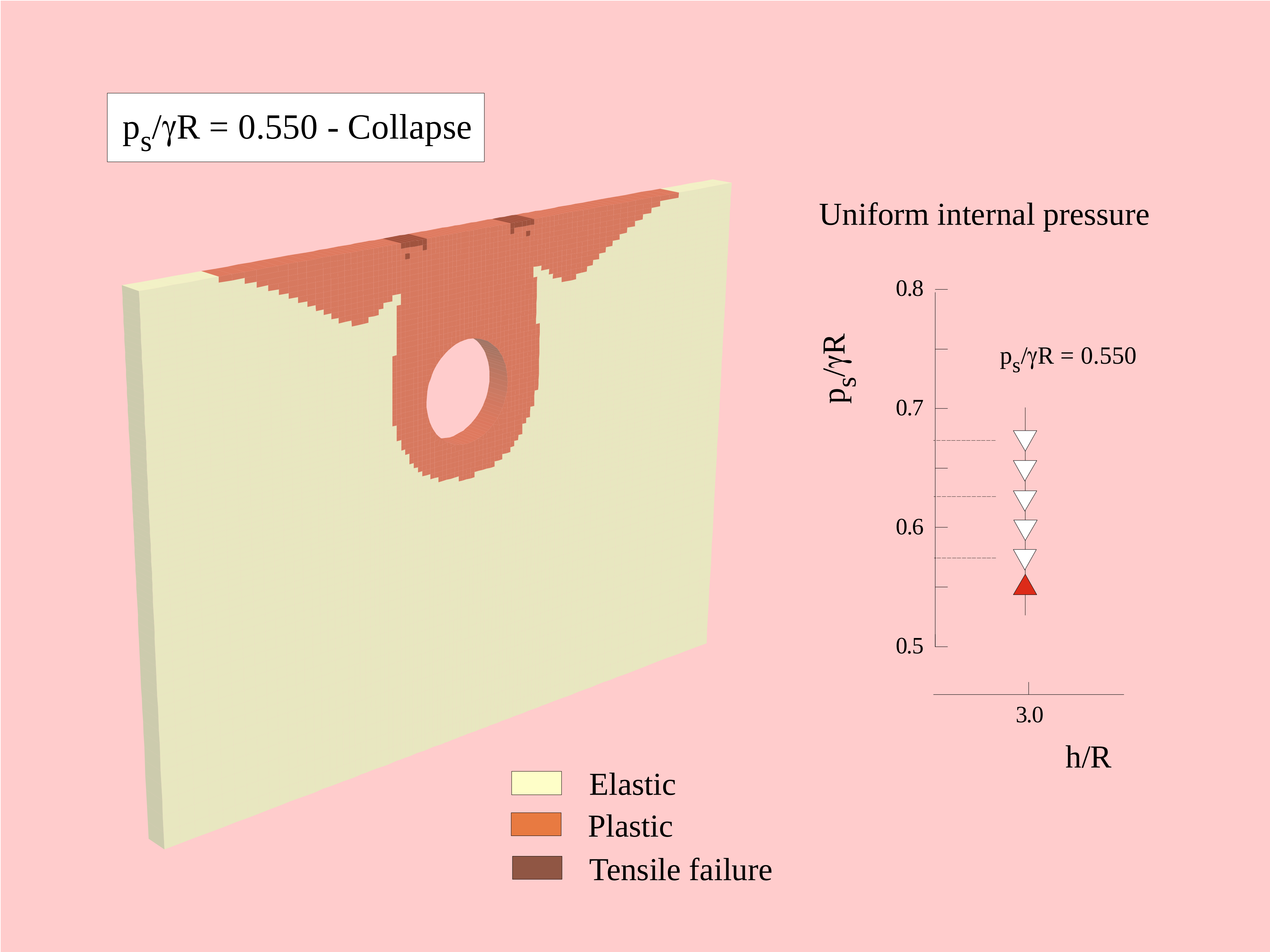
$p_s/\gamma R = 0.600$ - Equilibrium



Uniform internal pressure



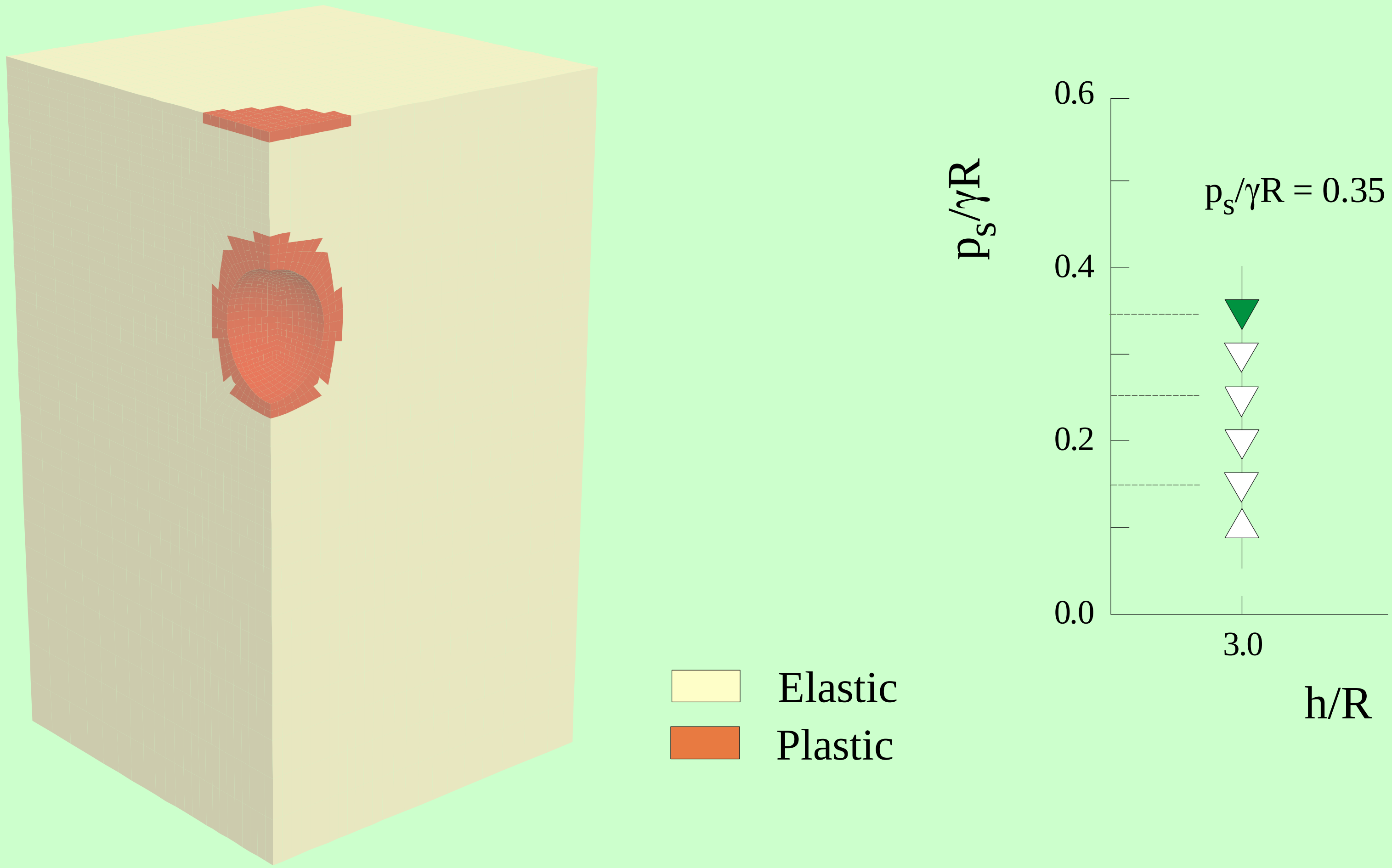
$p_s/\gamma R = 0.575$ - Equilibrium



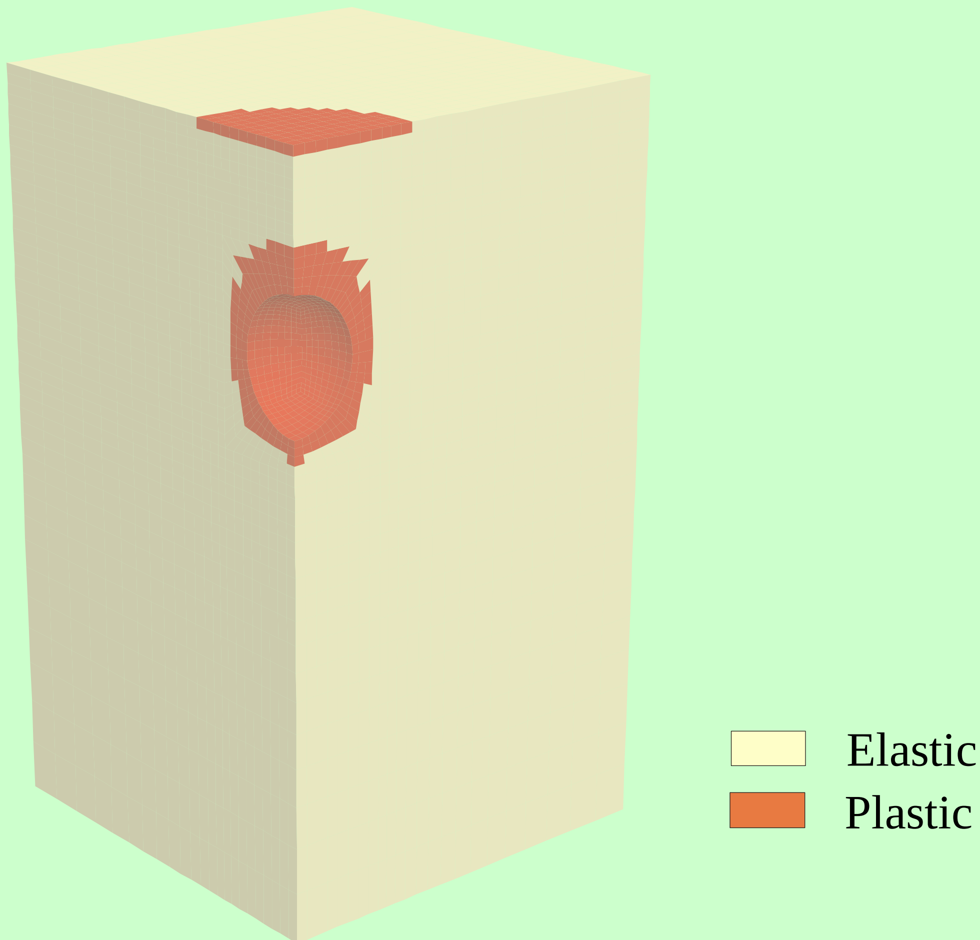
$p_s/\gamma R$ = 0.550 - Collapse
Uniform internal pressure
0.8
0.7
0.6
0.5
$p_s/\gamma R$
$p_s/\gamma R$ = 0.550
3.0
h/R
Elastic
Plastic
Tensile failure

$p_s/\gamma R = 0.35$ - Equilibrium

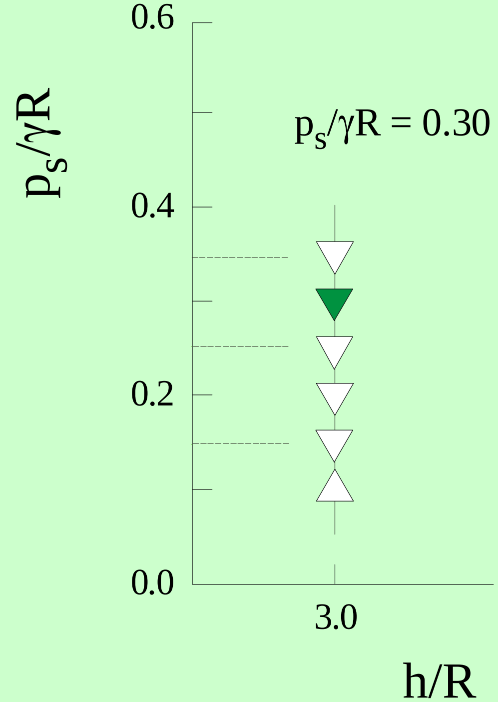
Uniform internal pressure

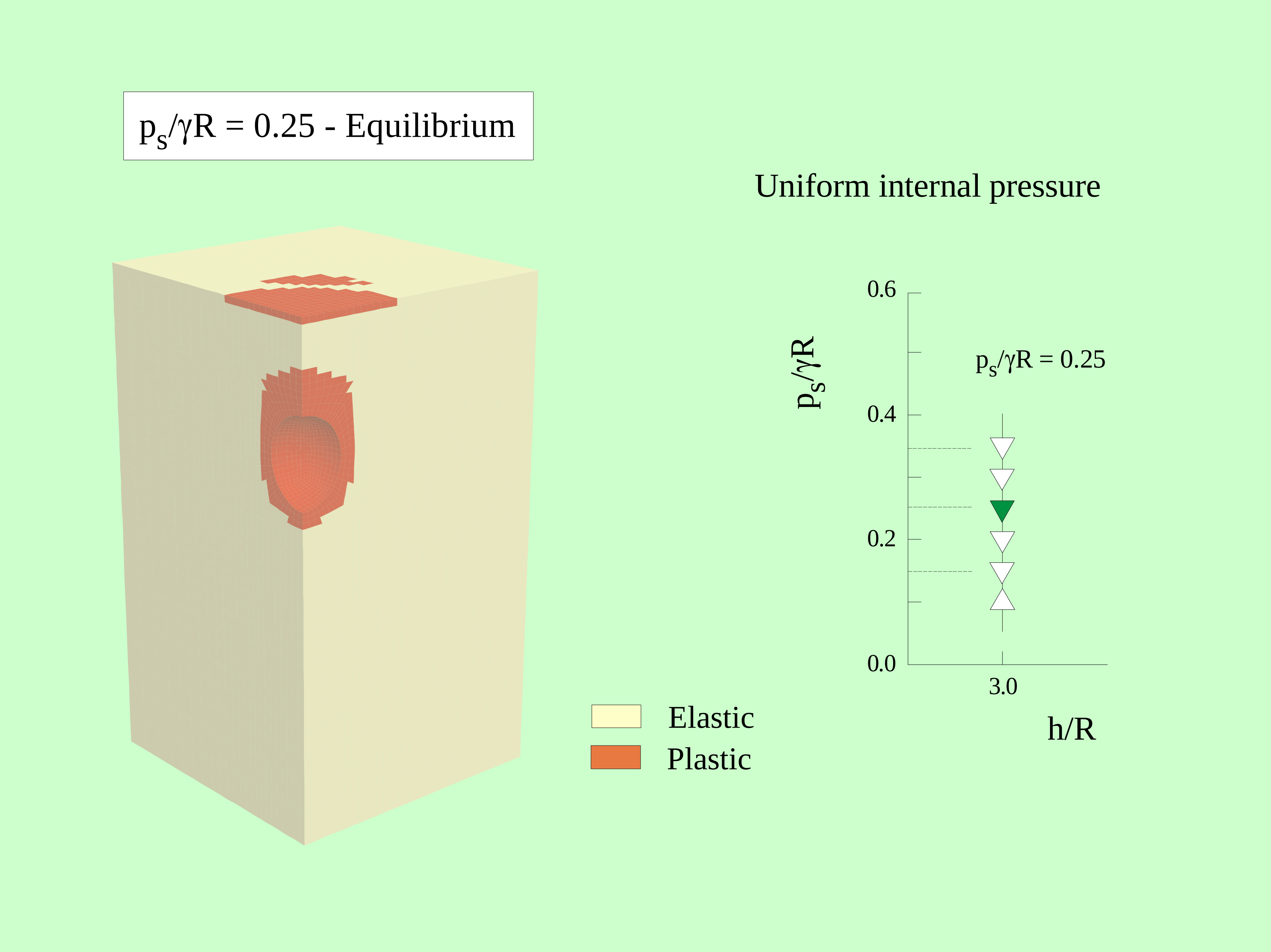


$p_s/\gamma R = 0.30$ - Equilibrium



Uniform internal pressure

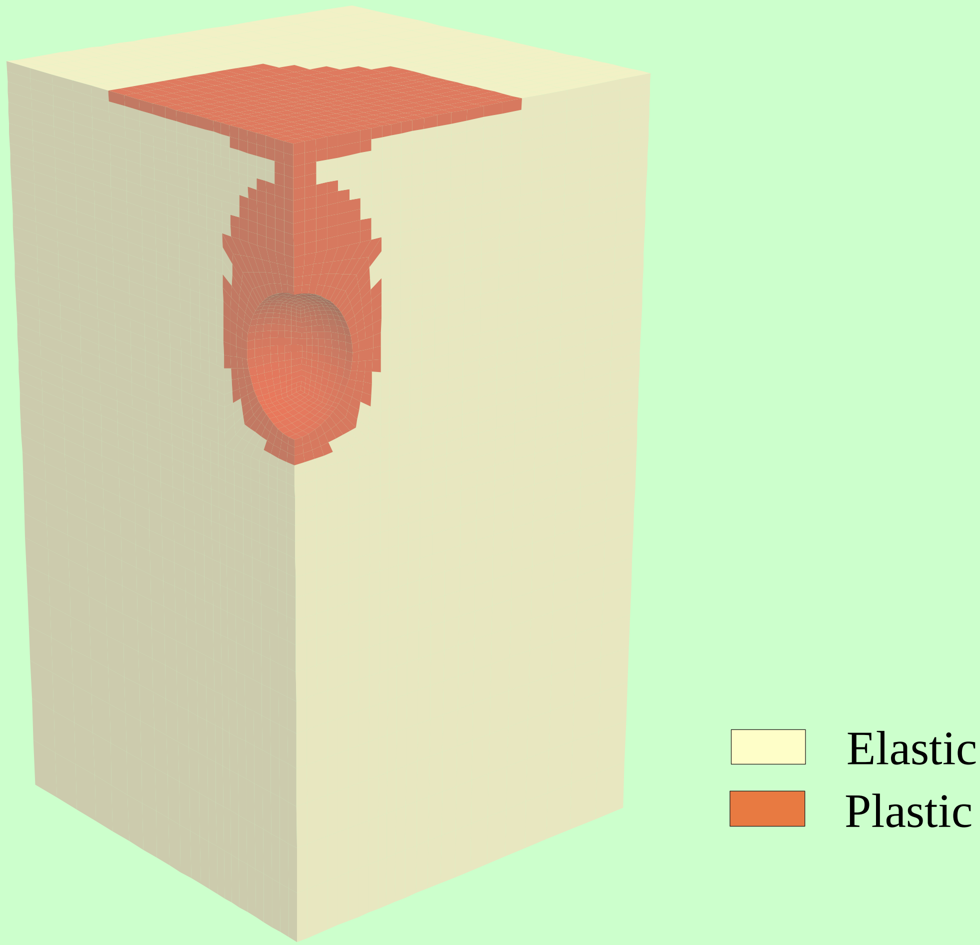


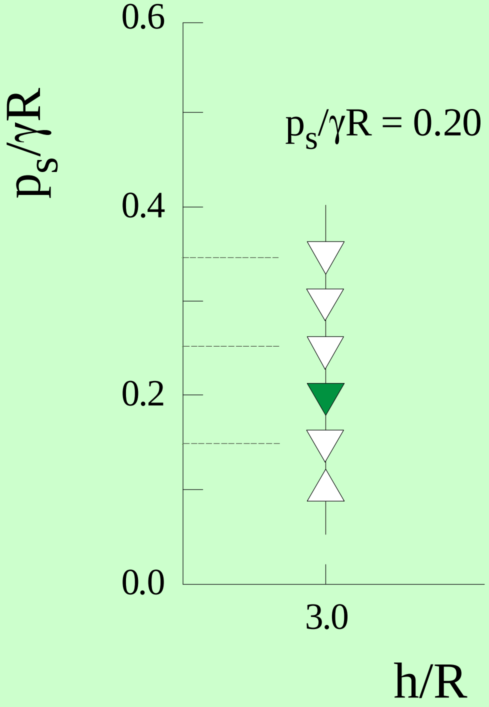


$p_s/\gamma R$ = 0.25 - Equilibrium
Uniform internal pressure
$p_s/\gamma R$
0.6
0.4
0.2
0.0
$p_s/\gamma R$ = 0.25
3.0
h/R
Elastic
Plastic

$p_s/\gamma R = 0.20$ - Equilibrium

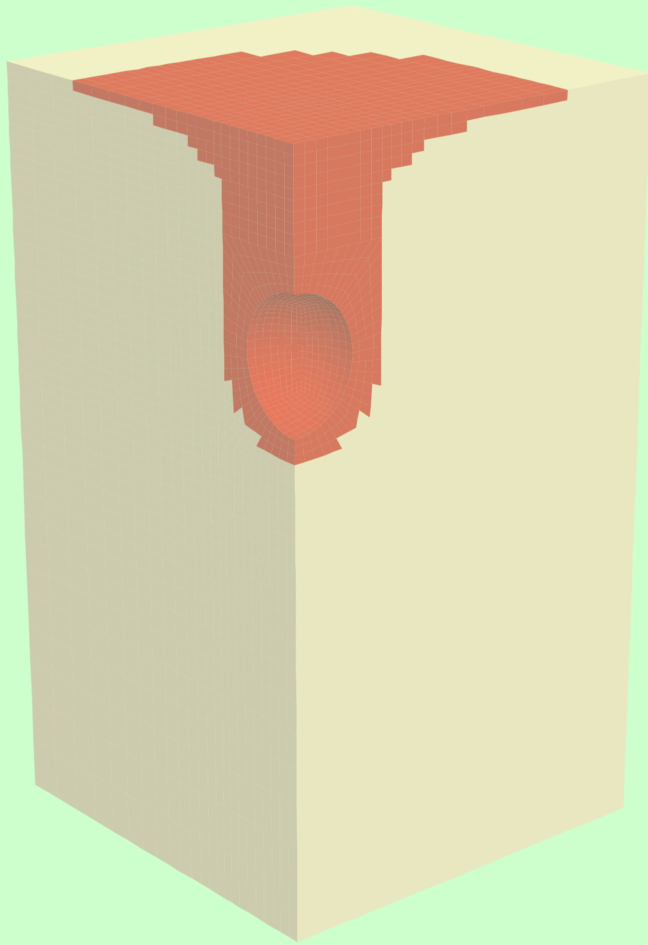
Uniform internal pressure





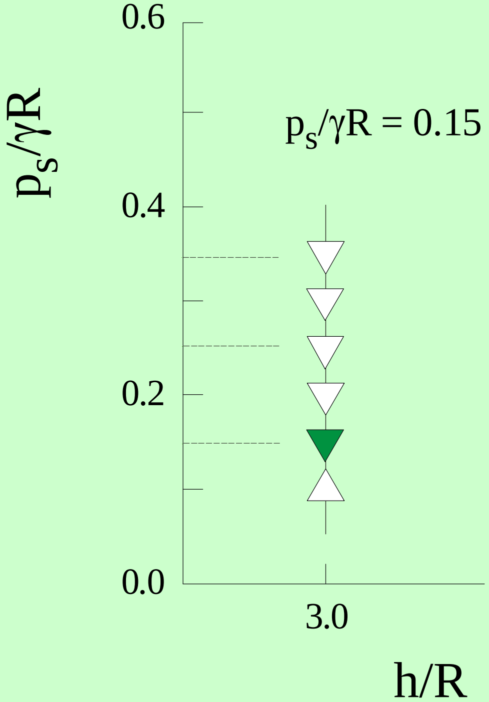
$p_s/\gamma R = 0.15$ - Equilibrium

Uniform internal pressure

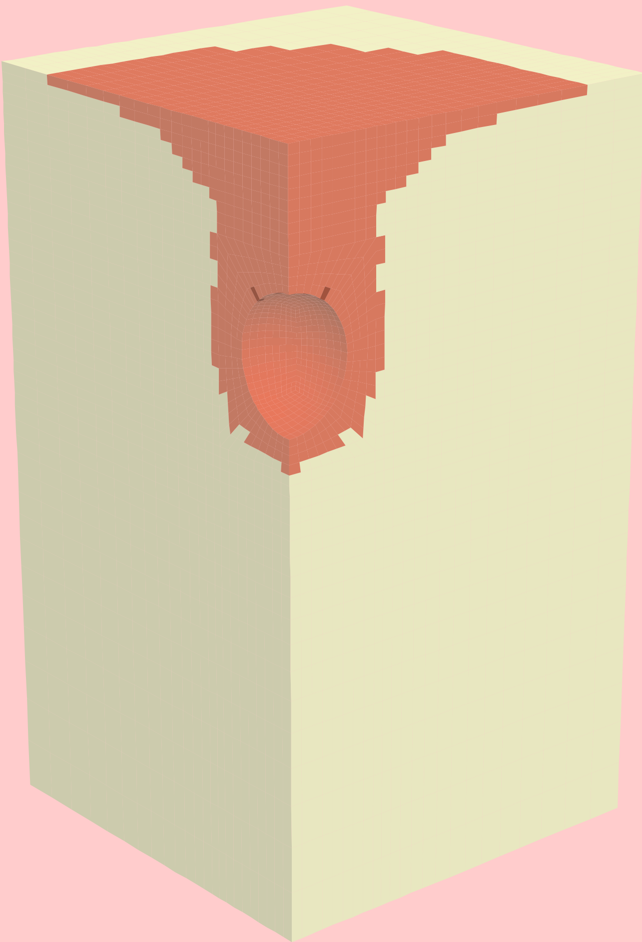


Elastic

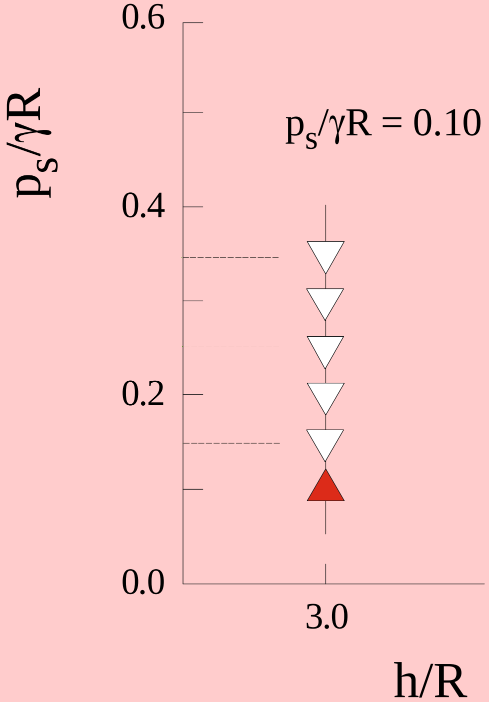
Plastic



$p_s/\gamma R = 0.10$ - Collapse



Uniform internal pressure



Elastic

Plastic

Tensile failure